第二章

直線運動

Motion along a straight line

Chapter 2

Straight Line Motion

本章授課重點

In this chapter we will study kinematics i.e. how objects move along a straight line.

The following parameters will be defined:

1. Displacement

2. Average velocity

3. Average Speed

4. Instantaneous velocity

5. Average and instantaneous acceleration

2.2 Motion

We find moving objects all around us.

The study of motion is called *kinematics*.

Examples:

- The Earth orbits around the Sun
- A roadway moves with Earth’s rotation

描述物體運動之時空觀念

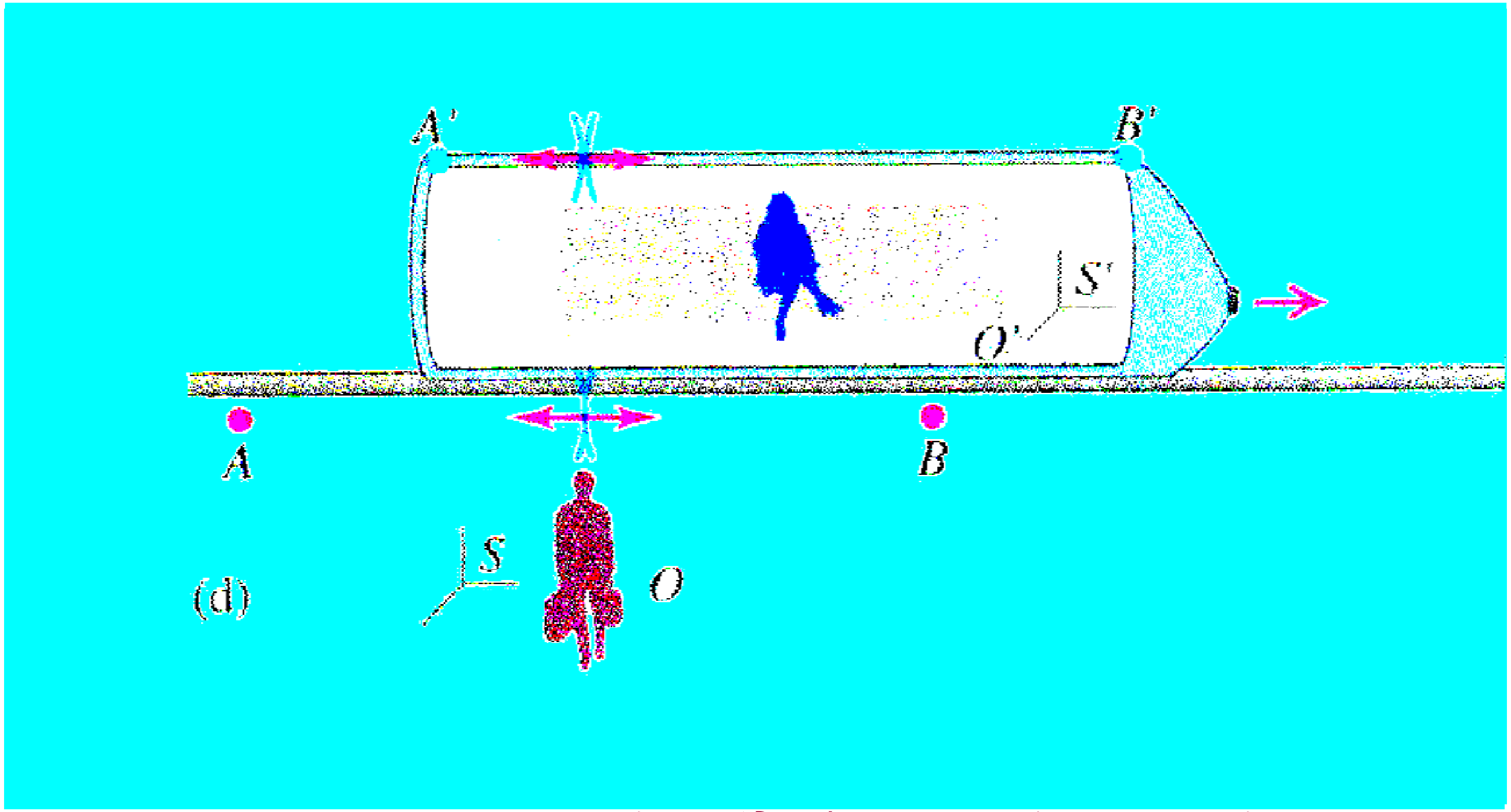
古典物理對時空的看法

- 空间是相對的
- 但時间是绝對的

狹義相對論對時空的看法

- 空间是相對的
- 且時间也是相對的

時間之相對性



同時----因不同的觀察者，而有不同的認知

為何從直線運動開始著手？

2.2 Motion

Here we will study motion that takes place in a straight line.

Forces cause motion. We will find out, as a result of application of force, if the objects speed up, slow down, or maintain the same rate.

The moving object here will be considered as a particle. If we deal with a stiff, extended object, we will assume that all particles on the body move in the same fashion. We will study the motion of a particle, which will represent the entire body.

2.3 Position and displacement

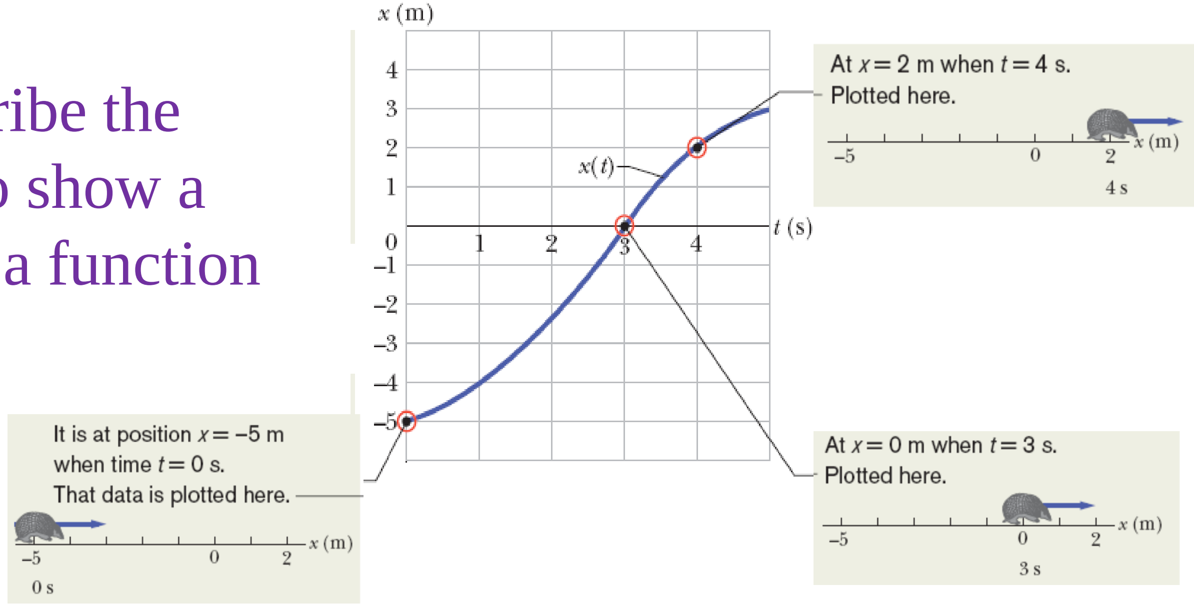
The location of an object is usually given in terms of a standard reference point, called the origin. The positive direction is taken to be the direction where the coordinates are increasing, and the negative direction as that where the coordinates are decreasing.

A change in the coordinates of the position of the body describes the *displacement* of the body.
For example, if the x-coordinate of a body changes from x_1 to x_2, then the displacement, $\Delta x = (x_2 - x_1)$.

Displacement is a vector quantity. That is, a quantity that has both magnitude and direction information.
An object's displacement is $x = -4$ m means that the object has moved towards decreasing x-axis by 4 m. The direction of motion, here, is toward decreasing x.

2.4 Average Velocity and Average Speed

A common way to describe the motion of an object is to show a graph of the position as a function of time.



Average velocity, or v_{avg}, is defined as the displacement over the time duration.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

The average velocity has the same sign as the displacement

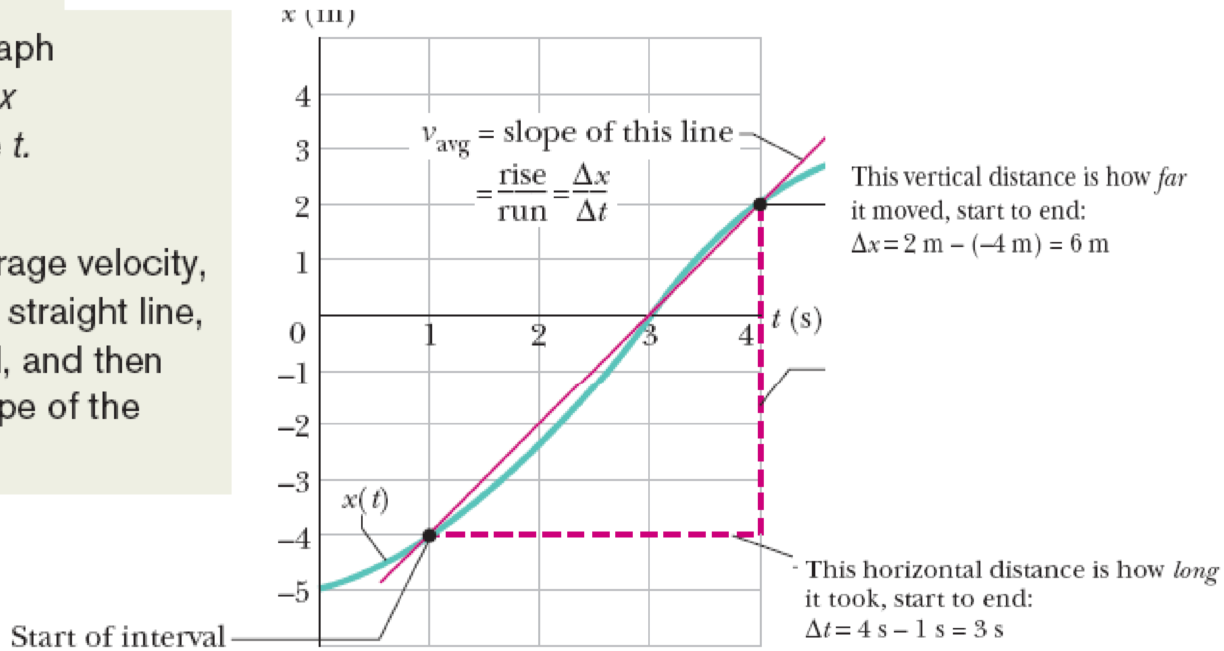
The magnitude of the slope of the x-t graph gives the average velocity

Here, the average velocity is:

$$v_{avg} = \frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}.$$

This is a graph of position x versus time t.

To find average velocity, first draw a straight line, start to end, and then find the slope of the line.



2.4 Average Speed

Average speed is the ratio of the total distance traveled to the total time duration. It is a scalar quantity, and does not carry any sense of direction.

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}.$$

總行走距離？！

Example, motion:

You drive a beat-up pickup truck along a straight road for 8.4 km at 70 km/h, at which point the truck runs out of gasoline and stops. Over the next 30 min, you walk another 2.0 km farther along the road to a gasoline station.

(a) What is your overall displacement from the beginning of your drive to your arrival at the station?

KEY IDEA

Assume, for convenience, that you move in the positive direction of an x axis, from a first position of $x_1 = 0$ to a second position of x_2 at the station. That second position must be at $x_2 = 8.4\text{ km} + 2.0\text{ km} = 10.4\text{ km}$. Then your displacement Δx along the x axis is the second position minus the first position.

$$\Delta x = x_2 - x_1 = 10.4\text{ km} - 0 = 10.4\text{ km}. \qquad \text{(Answer)}$$

Thus, your overall displacement is 10.4 km in the positive direction of the x axis.

Example, motion:

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

KEY IDEA

We already know the walking time interval Δt_{wlk} (= 0.50 h), but we lack the driving time interval Δt_{dr}. However, we know that for the drive the displacement Δx_{dr} is 8.4 km and the average velocity $v_{avg,dr}$ is 70 km/h. Thus, this average velocity is the ratio of the displacement for the drive to the time interval for the drive.

Calculations: We first write

$$v_{avg,dr} = \frac{\Delta x_{dr}}{\Delta t_{dr}}.$$

Rearranging and substituting data then give us

$$\Delta t_{dr} = \frac{\Delta x_{dr}}{v_{avg,dr}} = \frac{8.4 \text{ km}}{70 \text{ km/h}} = 0.12 \text{ h}.$$

So,
$$\Delta t = \Delta t_{dr} + \Delta t_{wlk}$$
$$= 0.12 \text{ h} + 0.50 \text{ h} = 0.62 \text{ h}. \qquad \text{(Answer)}$$

Example, motion:

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station? Find it both numerically and graphically.

KEY IDEA

From Eq. 2-2 we know that v_{avg} *for the entire trip* is the ratio of the displacement of 10.4 km *for the entire trip* to the time interval of 0.62 h *for the entire trip.*

Calculation: Here we find

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{10.4 \text{ km}}{0.62 \text{ h}}$$

$$= 16.8 \text{ km/h} \approx 17 \text{ km/h}. \qquad \text{(Answer)}$$

To find v_{avg} graphically, first we graph the function $x(t)$ as shown in Fig. 2-5, where the beginning and arrival points on the graph are the origin and the point labeled as "Station." Your average velocity is the slope of the straight line connecting those points; that is, v_{avg} is the ratio of the *rise* ($\Delta x = 10.4$ km) to the *run* ($\Delta t = 0.62$ h), which gives us $v_{avg} = 16.8$ km/h.

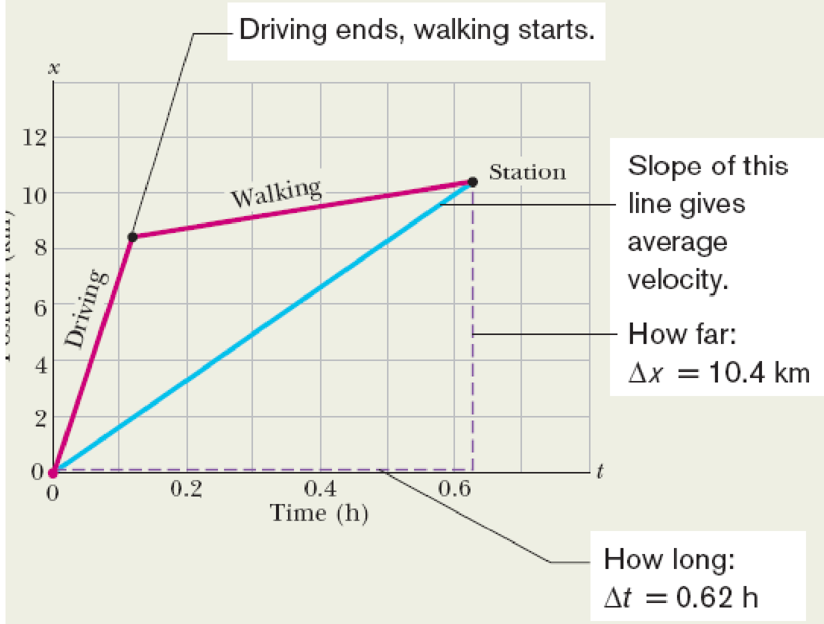
(d) Suppose that to pump the gasoline, pay for it, and walk back to the truck takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the gasoline?

KEY IDEA

Your average speed is the ratio of the total distance you move to the total time interval you take to make that move.

Calculation: The total distance is 8.4 km + 2.0 km + 2.0 km = 12.4 km. The total time interval is 0.12 h + 0.50 h + 0.75 h = 1.37 h. Thus, Eq. 2-3 gives us

$$s_{avg} = \frac{12.4 \text{ km}}{1.37 \text{ h}} = 9.1 \text{ km/h}. \qquad \text{(Answer)}$$



思考題

某人欲從甲開車至乙地，且希望能以40 km/hr的平均時速完成其行程。但當此人恰開至路途的一半時，方才發覺他一直是以平均時速20 km/hr的速率在行駛中。問剩下之路程，此人需以多少平均時速行駛，方能達其所願?

(A) 60 km/hr
(B) 90 km/hr
(C) 120 km/hr
(D) 不可能如期所願。

2.5: Instantaneous Velocity and Speed

The instantaneous velocity of a particle at a particular instant is the velocity of the particle at that instant.

Here Δt approaches a limiting value:

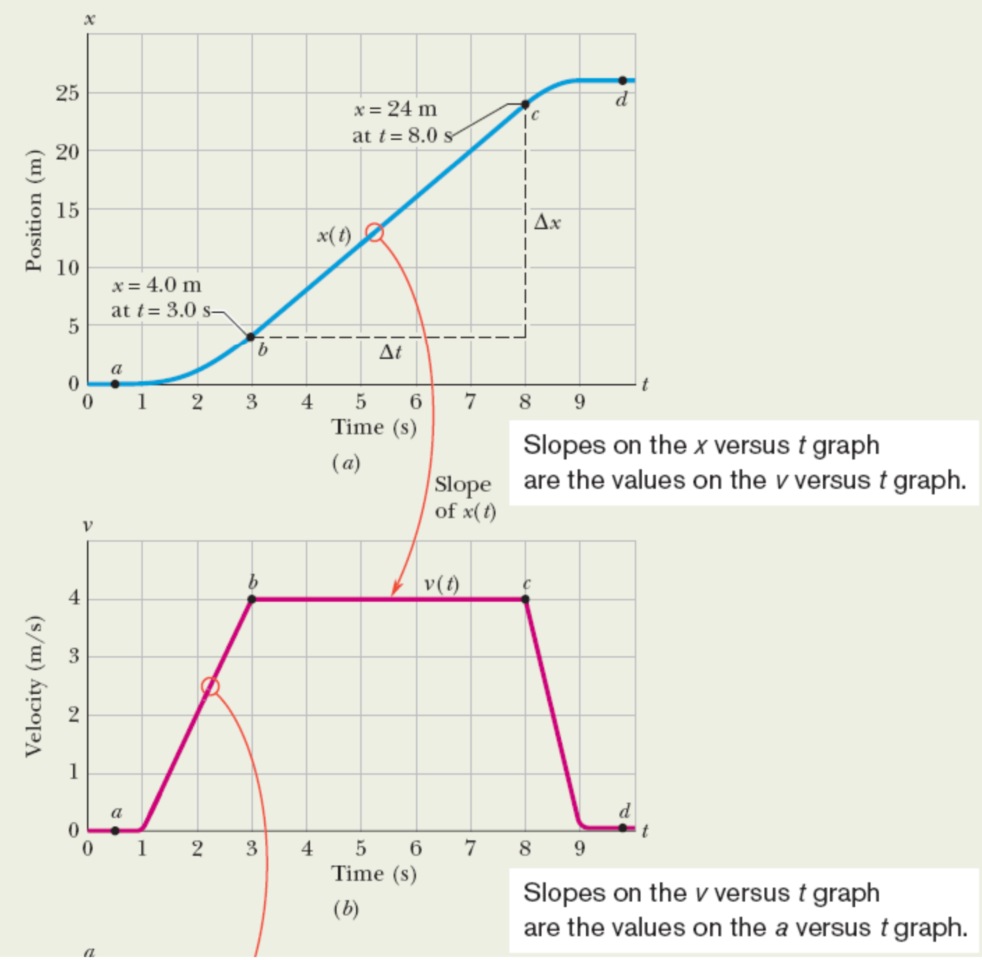
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

v, the instantaneous velocity, is the slope of the tangent of the position-time graph at that particular instant of time.

Velocity is a vector quantity and has with it an associated sense of direction.

Example, instantaneous velocity:

Figure 2-6*a* is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot $v(t)$.



KEY IDEA

We can find the velocity at any time from the slope of the $x(t)$ curve at that time.

Calculations: The slope of $x(t)$, and so also the velocity, is zero in the intervals from 0 to 1 s and from 9 s on, so then the cab is stationary. During the interval *bc*, the slope is constant and nonzero, so then the cab moves with constant velocity. We calculate the slope of $x(t)$ then as

$$\frac{\Delta x}{\Delta t} = v = \frac{24 \text{ m} - 4.0 \text{ m}}{8.0 \text{ s} - 3.0 \text{ s}} = +4.0 \text{ m/s}. \qquad (2\text{-}5)$$

The plus sign indicates that the cab is moving in the positive x direction. These intervals (where $v = 0$ and $v = 4$ m/s) are plotted in Fig. 2-6*b*. In addition, as the cab initially begins to move and then later slows to a stop, v varies as indicated in the intervals 1 s to 3 s and 8 s to 9 s. Thus, Fig. 2-6*b* is the required plot. (Figure 2-6*c* is considered in Section 2-6.)

Given a $v(t)$ graph such as Fig. 2-6*b*, we could "work backward" to produce the shape of the associated $x(t)$ graph (Fig. 2-6*a*). However, we would not know the actual values for x at various times, because the $v(t)$ graph indicates only *changes* in x. To find such a change in x during any interval, we must, in the language of calculus, calculate the area "under the curve" on the $v(t)$ graph for that interval. For example, during the interval 3 s to 8 s in which the cab has a velocity of 4.0 m/s, the change in x is

$$\Delta x = (4.0 \text{ m/s})(8.0 \text{ s} - 3.0 \text{ s}) = +20 \text{ m}. \qquad (2\text{-}6)$$

(This area is positive because the $v(t)$ curve is above the t axis.) Figure 2-6*a* shows that x does indeed increase by 20 m in that interval. However, Fig. 2-6*b* does not tell us the *values* of x at the beginning and end of the interval. For that, we need additional information, such as the value of x at some instant.

2.6: Average and instant accelerations

Average acceleration is the change of velocity over the change of time.
As such,

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

Here the velocity is v_1 at time t_1, and the velocity is v_2 at time t_2.

The instantaneous acceleration is defined as:

$$a = \frac{dv}{dt}.$$

In terms of the position function, the acceleration can be defined as:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

The SI units for acceleration are m/s^2.

2.6: Average and instant accelerations

If a particle has the same sign for velocity and acceleration, then that particle is speeding up.

Conversely, if a particle has opposite signs for the velocity and acceleration, then the particle is slowing down.

Our bodies often react to accelerations but not to velocities. A fast car often does not bother the rider, but a sudden brake is felt strongly by the rider. This is common in amusement car rides, where the rides change velocities quickly to thrill the riders.

The magnitude of acceleration falling near the Earth's surface is 9.8 m/s^2, and is often referred to as $\boldsymbol{g}$.

Colonel J. P. Stapp in a rocket sled, which undergoes sudden change in velocities.

Example, acceleration:

A particle's position on the x axis of Fig. 2-1 is given by

$$x = 4 - 27t + t^3,$$

with x in meters and t in seconds.

(a) Because position x depends on time t, the particle must be moving. Find the particle's velocity function $v(t)$ and acceleration function $a(t)$.

KEY IDEAS

(1) To get the velocity function $v(t)$, we differentiate the position function $x(t)$ with respect to time. (2) To get the acceleration function $a(t)$, we differentiate the velocity function $v(t)$ with respect to time.

Calculations: Differentiating the position function, we find

$$v = -27 + 3t^2, \qquad \text{(Answer)}$$

with v in meters per second. Differentiating the velocity function then gives us

$$a = +6t, \qquad \text{(Answer)}$$

with a in meters per second squared.

(b) Is there ever a time when $v = 0$?

Calculation: Setting $v(t) = 0$ yields

$$0 = -27 + 3t^2,$$

which has the solution

$$t = \pm 3 \text{ s}. \qquad \text{(Answer)}$$

Thus, the velocity is zero both 3 s before and 3 s after the clock reads 0.

(c) Describe the particle's motion for $t \geq 0$.

Reasoning: We need to examine the expressions for $x(t)$, $v(t)$, and $a(t)$.

At $t = 0$, the particle is at $x(0) = +4$ m and is moving with a velocity of $v(0) = -27$ m/s—that is, in the negative direction of the x axis. Its acceleration is $a(0) = 0$ because just then the particle's velocity is not changing.

For $0 < t < 3$ s, the particle still has a negative velocity, so it continues to move in the negative direction. However, its acceleration is no longer 0 but is increasing and positive. Because the signs of the velocity and the acceleration are opposite, the particle must be slowing.

Indeed, we already know that it stops momentarily at $t = 3$ s. Just then the particle is as far to the left of the origin in Fig. 2-1 as it will ever get. Substituting $t = 3$ s into the expression for $x(t)$, we find that the particle's position just then is $x = -50$ m. Its acceleration is still positive.

For $t > 3$ s, the particle moves to the right on the axis. Its acceleration remains positive and grows progressively larger in magnitude. The velocity is now positive, and it too grows progressively larger in magnitude.

2.7: Constant acceleration

When the acceleration is constant, its average and instantaneous values are the same.

$a = a_{\text{avg}} = \dfrac{v - v_0}{t - 0}$. means that $\boxed{v = v_0 + at.}$ ……(1)

Here, velocity at t=0 is v_o.

Similarly, $v_{\text{avg}} = \dfrac{x - x_0}{t - 0}$ which means that $x = x_0 + v_{\text{avg}} t,$

finally leading to $\boxed{x - x_0 = v_0 t + \frac{1}{2} a t^2.}$ …..(2)

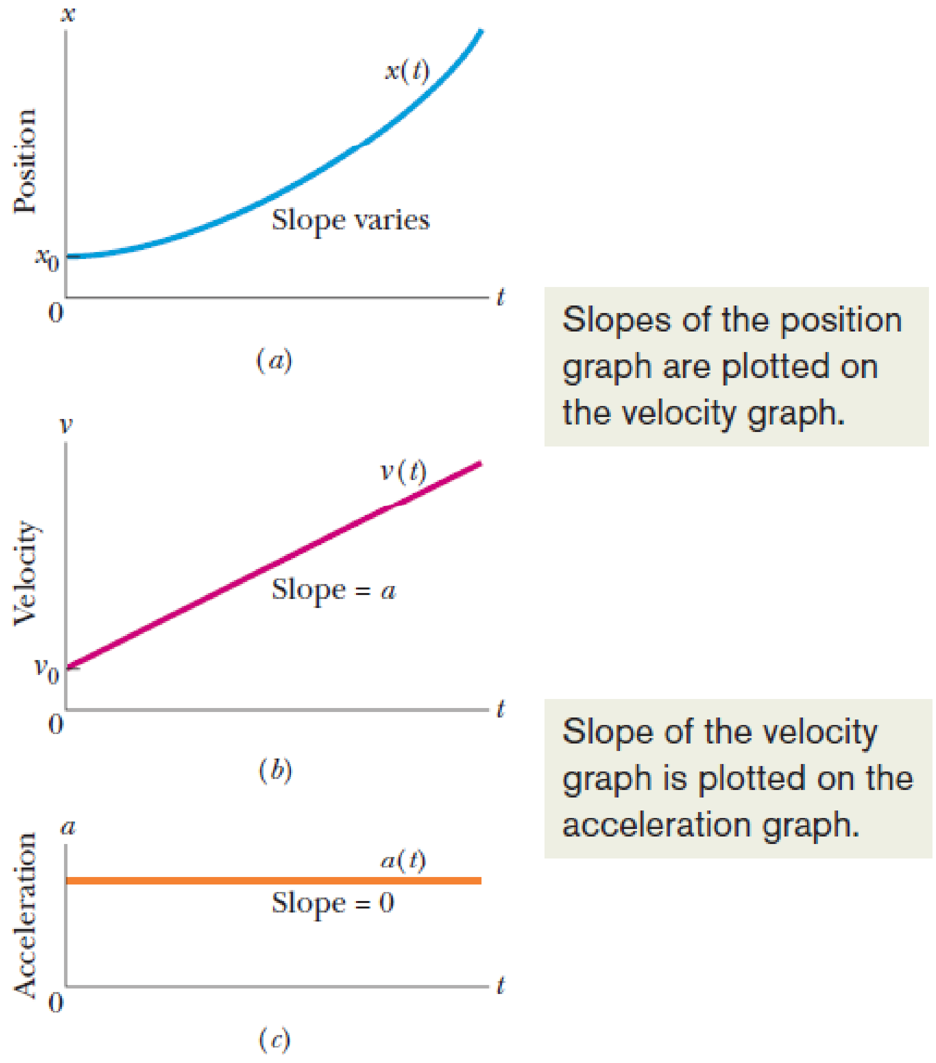
Eliminating t from the Equations (1) and (2):

$$\boxed{v^2 = v_0^2 + 2a(x - x_0).} \quad …..(3)$$

2.7: Constant acceleration

Integrating constant acceleration graph for a fixed time duration yields values for velocity graph during that time.

Similarly, integrating velocity graph will yield values for position graph.



Example, constant acceleration:

Figure 2-9 gives a particle's velocity v versus its position as it moves along an x axis with constant acceleration. What is its velocity at position $x = 0$?

KEY IDEA

We can use the constant-acceleration equations; in particular, we can use Eq. 2-16 ($v^2 = v_0^2 + 2a(x - x_0)$), which relates velocity and position.

First try: Normally we want to use an equation that includes the requested variable. In Eq. 2-16, we can identify x_0 as 0 and v_0 as being the requested variable. Then we can identify a second pair of values as being v and x. From the graph, we have

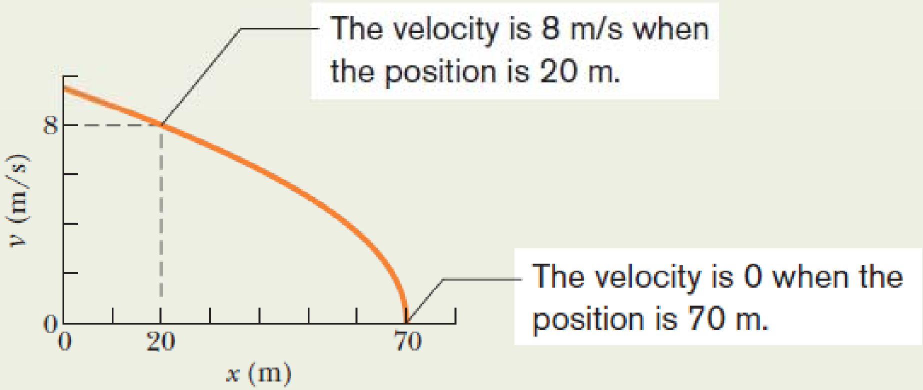


Fig. 2-9 Velocity versus position.

two such pairs: (1) $v = 8$ m/s and $x = 20$ m, and (2) $v = 0$ and $x = 70$ m. For example, we can write Eq. 2-16 as

$$(8\text{ m/s})^2 = v_0^2 + 2a(20\text{ m} - 0). \qquad (2\text{-}19)$$

However, we know neither v_0 nor a.

Second try: Instead of directly involving the requested variable, let's use Eq. 2-16 with the two pairs of known data, identifying $v_0 = 8$ m/s and $x_0 = 20$ m as the first pair and $v = 0$ m/s and $x = 70$ m as the second pair. Then we can write

$$(0\text{ m/s})^2 = (8\text{ m/s})^2 + 2a(70\text{ m} - 20\text{ m}),$$

which gives us $a = -0.64$ m/s^2. Substituting this value into Eq. 2-19 and solving for v_0 (the velocity associated with the position of $x = 0$), we find

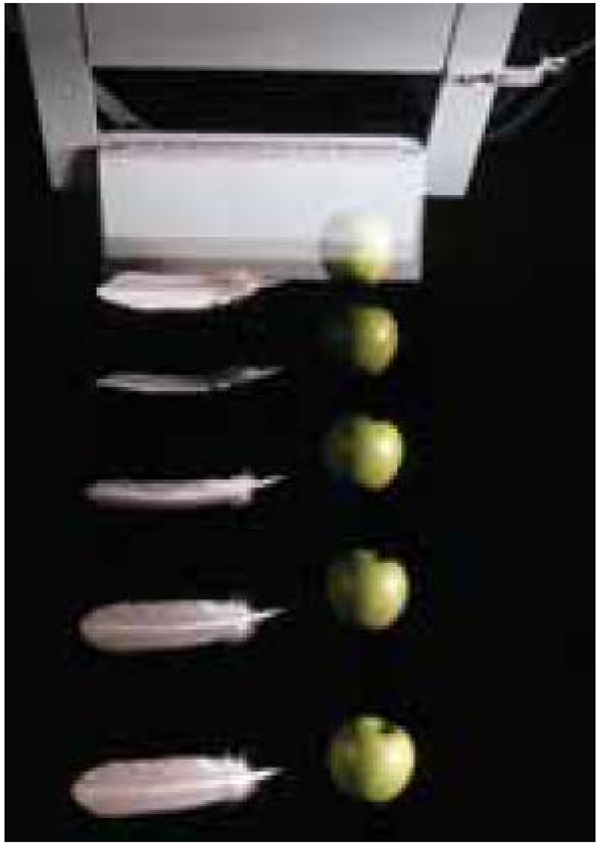
$$v_0 = 9.5\text{ m/s}. \qquad \text{(Answer)}$$

2.9: Free-Fall Acceleration

In this case objects close to the Earth's surface fall towards the Earth's surface with no external forces acting on them except for their weight.

Use the constant acceleration model with "a" replaced by "-g", where g = 9.8 m/s^2 for motion close to the Earth's surface.

In vacuum, a feather and an apple will fall at the same rate.



Sample problem

In Fig. 2-11, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

KEY IDEAS

(1) Once the ball leaves the pitcher and before it returns to his hand, its acceleration is the free-fall acceleration $a = -g$. Because this is constant, Table 2-1 applies to the motion. (2) The velocity v at the maximum height must be 0.

Calculation: Knowing v, a, and the initial velocity $v_0 = 12$ m/s, and seeking t, we solve Eq. 2-11, which contains those four variables. This yields

$$t = \frac{v - v_0}{a} = \frac{0 - 12\text{ m/s}}{-9.8\text{ m/s}^2} = 1.2\text{ s}. \qquad \text{(Answer)}$$

(b) What is the ball's maximum height above its release point?

Calculation: We can take the ball's release point to be $y_0 = 0$. We can then write Eq. 2-16 in y notation, set $y - y_0 = y$ and $v = 0$ (at the maximum height), and solve for y. We get

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 7.3\text{ m}. \qquad \text{(Answer)}$$

(c) How long does the ball take to reach a point 5.0 m above its release point?

Calculations: We know v_0, $a = -g$, and displacement $y - y_0 = 5.0$ m, and we want t, so we choose Eq. 2-15. Rewriting it for y and setting $y_0 = 0$ give us

$$y = v_0 t - \tfrac{1}{2}gt^2,$$

or

$$5.0\text{ m} = (12\text{ m/s})t - (\tfrac{1}{2})(9.8\text{ m/s}^2)t^2.$$

If we temporarily omit the units (having noted that they are consistent), we can rewrite this as

$$4.9t^2 - 12t + 5.0 = 0.$$

Solving this quadratic equation for t yields

$$t = 0.53\text{ s} \quad \text{and} \quad t = 1.9\text{ s}. \qquad \text{(Answer)}$$

There are two such times! This is not really surprising because the ball passes twice through $y = 5.0$ m, once on the way up and once on the way down.

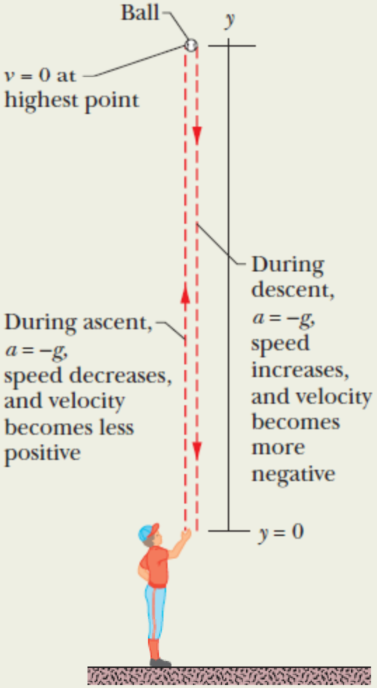


Fig. 2-11 A pitcher tosses a baseball straight up into the air. The equations of free fall apply for rising as well as for falling objects, provided any effects from the air can be neglected.

2-10: Graphical integration in motion analysis

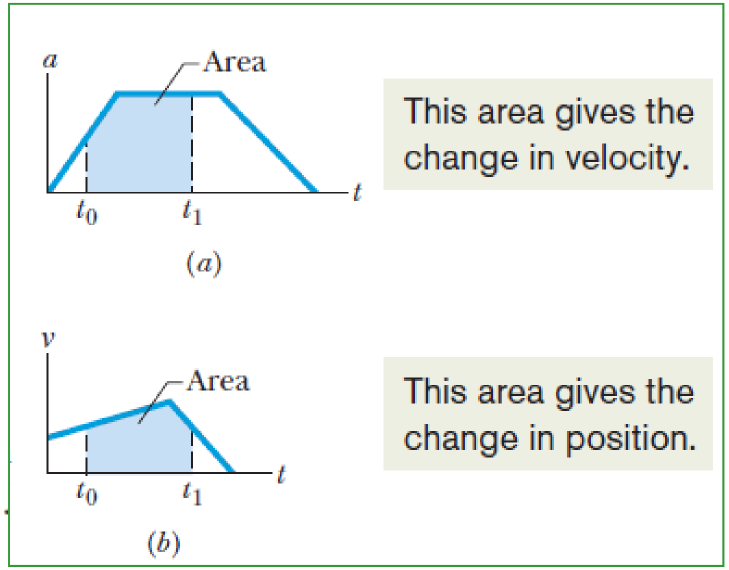
Starting from

$$a = dv/dt$$

we obtain

$$v_1 - v_0 = \int_{t_0}^{t_1} a\, dt.$$

(v_o= velocity at time t=0, and v_1= velocity at time t = t_1).



Note that

$$\int_{t_0}^{t_1} a\, dt = \begin{pmatrix}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

Similarly, we obtain

$$x_1 - x_0 = \int_{t_0}^{t_1} v\, dt,$$

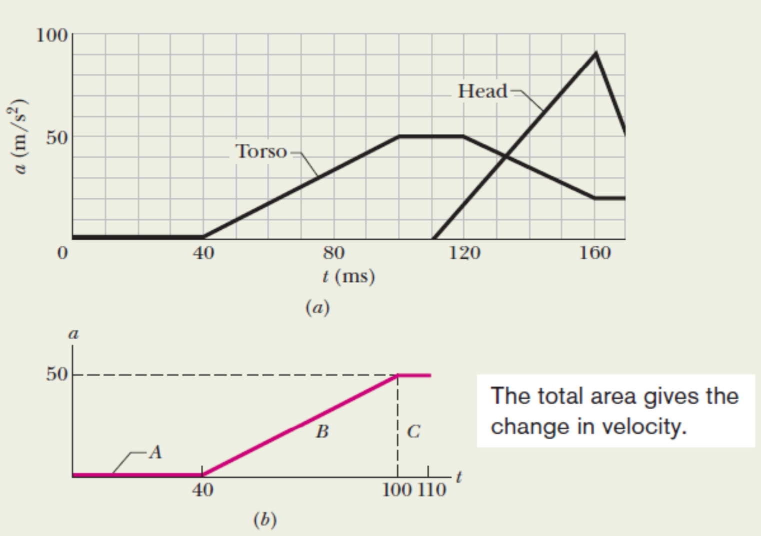
(x_o= position at time t = 0, and x_1 = position at time t=t_1), and

$$\int_{t_0}^{t_1} v\, dt = \begin{pmatrix}\text{area between velocity curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{pmatrix}.$$

Example, graphical solution:

"Whiplash injury" commonly occurs in a rear-end collision where a front car is hit from behind by a second car. In the 1970s, researchers concluded that the injury was due to the occupant's head being whipped back over the top of the seat as the car was slammed forward. As a result of this finding, head restraints were built into cars, yet neck injuries in rear-end collisions continued to occur.

In a recent test to study neck injury in rear-end collisions, a volunteer was strapped to a seat that was then moved abruptly to simulate a collision by a rear car moving at 10.5 km/h. Figure 2-13*a* gives the accelerations of the volunteer's torso and head during the collision, which began at time $t = 0$. The torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the volunteer. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?



KEY IDEA

We can calculate the torso speed at any time by finding an area on the torso $a(t)$ graph.

Calculations: We know that the initial torso speed is $v_0 = 0$ at time $t_0 = 0$, at the start of the "collision." We want the torso speed v_1 at time $t_1 = 110$ ms, which is when the head begins to accelerate.

$$v_1 - v_0 = \left(\begin{array}{c}\text{area between acceleration curve}\\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right).$$

For convenience, let us separate the area into three regions (Fig. 2-13*b*). From 0 to 40 ms, region *A* has no area:

$$\text{area}_A = 0.$$

From 40 ms to 100 ms, region *B* has the shape of a triangle, with area

$$\text{area}_B = \tfrac{1}{2}(0.060\text{ s})(50\text{ m/s}^2) = 1.5\text{ m/s}.$$

From 100 ms to 110 ms, region *C* has the shape of a rectangle, with area

$$\text{area}_C = (0.010\text{ s})(50\text{ m/s}^2) = 0.50\text{ m/s}.$$

Substituting these values and $v_0 = 0$ into Eq. 2-26 gives us

$$v_1 - 0 = 0 + 1.5\text{ m/s} + 0.50\text{ m/s},$$

or

$$v_1 = 2.0\text{ m/s} = 7.2\text{ km/h}. \qquad \text{(Answer)}$$

解題主要重點提示

距離

移動長度

位移

大小
不論方向
(純量)

區分距離與位移分別
區分速率與速度分別

大小
和方向
(向量)

速率

須知時間和移動長度

速度

$$\vec{v}(t)=\frac{d\vec{r}(t)}{dt}$$

$$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=\frac{d^2\vec{r}(t)}{dt^2}$$

位移 $\vec{r}(t)$

微分

積分

速度 $\vec{v}(t)$

微分

積分

加速度 $\vec{a}(t)$

$$\vec{r}(t)-\vec{r}_0=\int_{t=0}^{t}\vec{v}(t)dt=\vec{v}_0t+\int_{t=0}^{t}dt\int_{t=0}^{t}\vec{a}(t)dt$$

$$\vec{v}(t)-\vec{v}_0=\int_{t=0}^{t}\vec{a}(t)dt$$

8th Ed【CH02】Straight Line Motion

9th Ed【CH02】Motion along a Straight Line

（★指的是題目裡面數字有改！原則上，目前習題解答以 8th Ed 題目爲準。）

8th Ed：Homework of Chapter 2：

5, 9, 13, 15, 17, 19, 21, 25,27,40, 41, 47, 49,51, 57, 63

8h Ed【Problem 2-5】: 9^{th} Ed【Problem 2-5】

The position of an object moving along an x axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds. Find the position of the object at the following values of t: (a) 1s, (b) 2s, (c) 3s, and (d) 4s. (e) What is the object's displacement between $t = 0$ and $t = 4s$? (f) What is average velocity for the time interval from $t = 2s$ to $t = 4s$? (g) graph x versus t fro $0 \le t \le 4s$ and indicate how the answer for (f) can be found on the graph.

一沿直線運動的物體，其位置爲 $x = 3t - 4t^2 + t^3$（x 單位爲米，t 單位爲秒），找出當時間爲（a）1 秒（b）2 秒（c）3 秒和（d）4 秒時的位置。（e）時間在 0 到 4 秒間，位移爲何？（f）時間在 2 到 4 秒間，平均速度爲何（g）畫出 $0 \le t \le 4s$ 間的 x-t 圖，由圖說明（f）的答案。

<解>：(a) $x = 3(1) - 4(1)^2 + (1)^3 = 0$

(b) $x = 3(2) - 4(2)^2 + (2)^3 = -2$

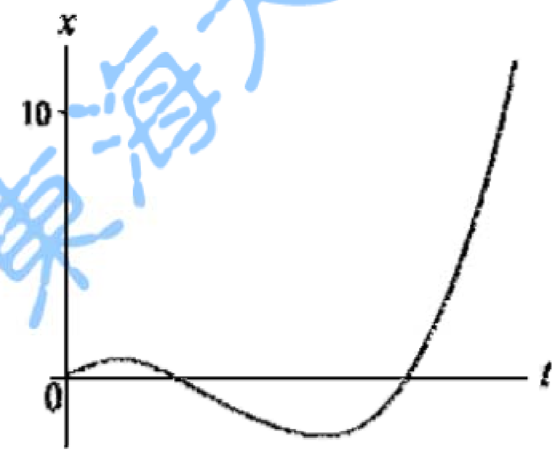
(c) $x = 3(3) - 4(3)^2 + (3)^3 = 0$

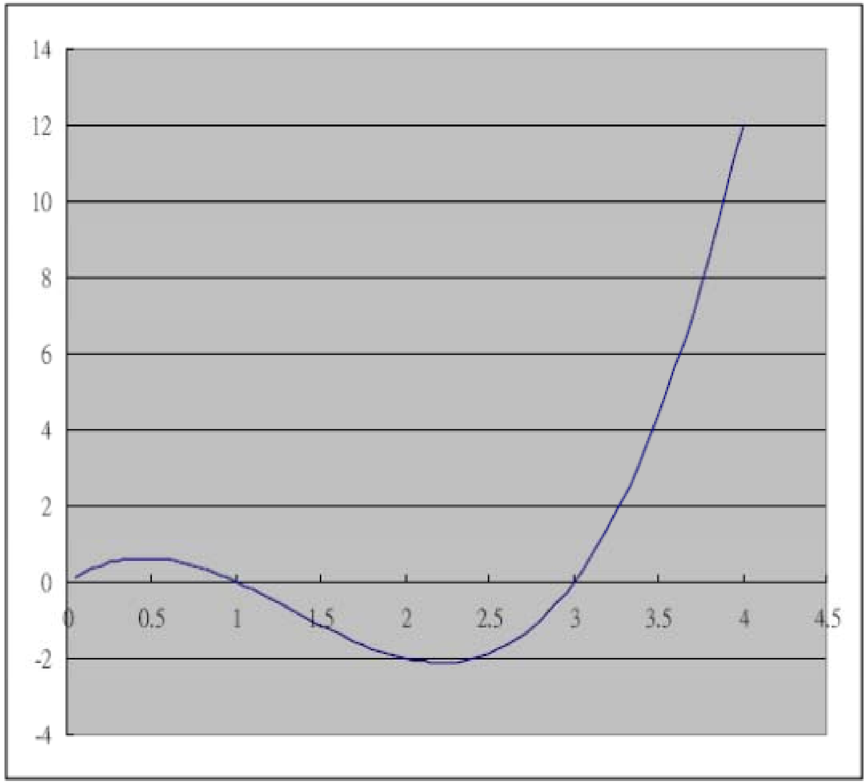
(d) $x = 3(4) - 4(4)^2 + (4)^3 = 12$

(e) 12

(f) $v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{12 - (-2)}{2} = 7m/s$

(g) 畫圖

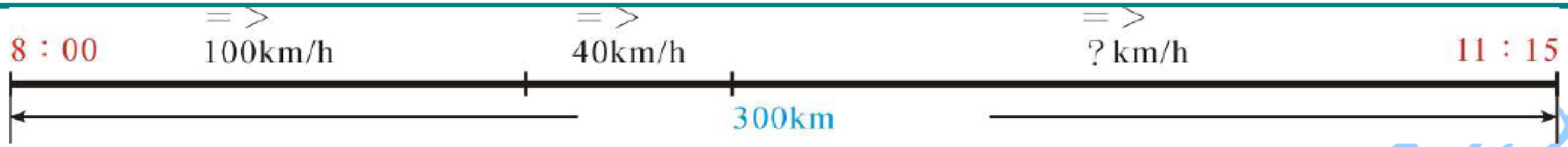




8th Ed【Problem 2-9】：9th Ed【Problem 2-11】

You are to drive to an interview in another town, at a distance of 300km on an expressway. The interview is at 11:15 A.M. you plan to drive at 100km/h, so you leave at 8:00 A.M. to allow some extra time. You drive at that speed for the first 100km, but then construction work forces you to slow to 40km/h for 40km. what would be the least speed needed for the rest of the trip to arrive in time for the interview?

你要開車到距離 300km 外的小鎮面試，面試時間爲早上 11：15。計畫速度 100km/h，打算早上 8 點出發，沒想到開了 100km 後遇到公路施工，速度降爲 40km/h 走了 40km。問最後一段距離你要開多快，面試才不會遲到？



<解>：1、第一段花的時間 $= \dfrac{100km}{100km/h} = 1h$

此時時間已經是 9：00AM

2、第二段花的時間 $= \dfrac{40km}{40km/h} = 1h$

此時時間已經是 10：00AM

3、剩下距離 $= 300km - 100km - 40km = 160km$

11：15AM 前要到，所以剩下時間 $= 1.25h$

速度 $= \dfrac{160km}{1.25h} = 128km/h$

8th Ed【Problem 2-13】: 9th Ed【Problem 2-13】

You drive on Interstate 10 from San Antonio to Houston, half the time at 55km/h and the other half at 90km/h. On the way back you travel half the distance at 55km/h and the other half at 90km/h. what is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch x versus t for (a), assuming the motion is all in the positive x direction. Indicate how the average velocity can be found on the sketch.

你開車行駛在的 10 號州際公路上，從聖安東尼奧要到休斯頓，旅程一半的時間，以 55km/h的速度行駛，另一半的時間以 90km/h 行駛。回程時，總路程一半的距離以 55km/h 時速行駛，另一半距離的路程以 90km/h 行駛。問平均速率 (a)從聖安東尼奧到休斯頓，(b) 從休斯頓到聖安東尼奧，(c)整個行程，(d)整個行程的平均速度，(e)畫出(a)中 x-t 圖。由圖上指出平均速度。

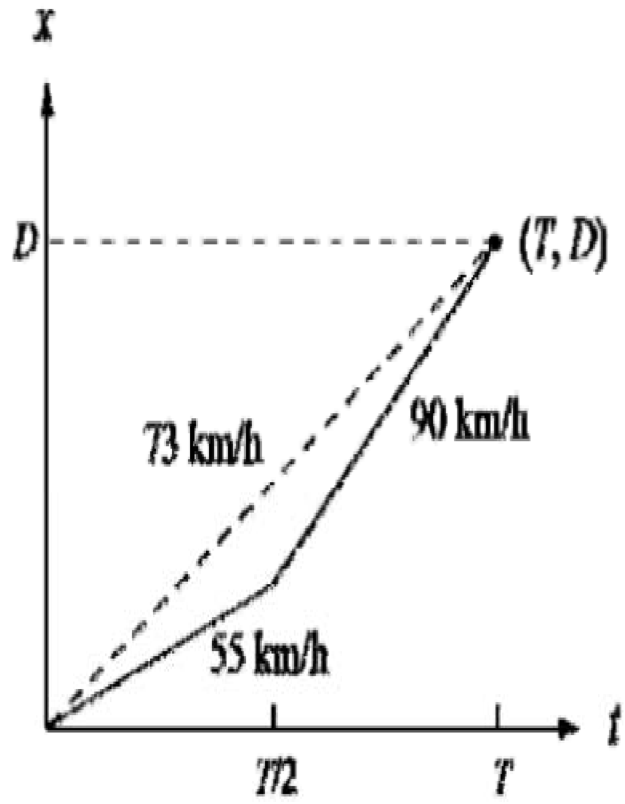
<解>：(a) $s_{av}=\dfrac{55\times\dfrac{T}{2}+90\times\dfrac{T}{2}}{T}=72.5km/h$

(b) $s_{av}=\dfrac{D}{\dfrac{D/2}{55}+\dfrac{D/2}{90}}=68.3km/h$

(c) $s_{av}=\dfrac{2D}{\dfrac{D}{72.5}+\dfrac{D}{68.3}}=70km/h$

(d) 0

(e)



8^{th} Ed【Problem 2-15】: 9^{th} Ed【Problem 2-15】

(a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what its velocity at t=1s? (b) Is it moving in the positive or negative direction of x just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time t; if not, answer no. (f) Is there a time after $t = 3s$ when the particle is moving in the negative direction of x? If so, given the time t; if not, answer no.

（a）假設一質點位置爲 $x = 4 - 12t + 3t^2$（t 單位爲秒，x 單位爲米）在 t 爲 1 秒時，問速度爲何？（b）此時，它是朝正 x 或負 x 方向移動？（c）此時速率爲何？（d）之後速度是增加或減少？(接下來兩個問題不需計算請直接回答)（e）有哪一瞬間速度爲零嗎？（f）在 $t = 3s$ 後，質點會沿負方向運動嗎？如果會，問何時？如果不會，直接回答不會。

<解>：(a) $v = -12 + 6t\,m/s$

$v(t = 1s) = -12 + 6(1) = -6m/s$

(b) 負方向

(c) speed $|v| = 6m/s$

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) $t = 2s$ 時，$v = 0$

(f) $v = -12 + 6t\,m/s$，$t > 2s$ 時，$v > 0$ 不會(因大於2秒後，速度皆爲正)

8th Ed【Problem 2-17】: 9th Ed【Problem 2-17】

The position of a particle moving along x axis is given in centimeter by $x = 9.75 + 1.50t^3$, where t is in second. Calculate (a) the average velocity during the time interval $t = 2s$ to $t = 3s$; (b) the instantaneous velocity at $t = 2s$; (c) the instantaneous velocity at $t = 3s$; (d) the instantaneous velocity at $t = 2.5s$; and (e) the instantaneous velocity when the particle is midway between its position at $t = 2s$ and $t = 3s$. (f) Graph x versus t and indicate your graphically.

一個粒子沿著 x 軸移動，其位置爲 $x = 9.75 + 1.50t^3$（t 單位爲秒，x 單位爲公分）。計算（a）在時間爲 2-3 秒間的平均速度。（b）在 2 秒時的瞬時速度。（c）在 3 秒時的瞬時速度。（d）在 2.5 秒時的瞬時速度。（e）在時間 2-3 秒時，粒子跑到中間時的瞬時速度。（f）畫出 x-t 圖。

<解>：(a) $v_{av} = \dfrac{[9.75 + 1.5(3^3)] - [9.75 + 1.5(2^3)]}{3s - 2s} = 28.5cm/s$

(b) $v = 4.5t^2$

$v(t = 2s) = 4.5(2)^2 = 18cm/s$

(c) $v(t = 3s) = 4.5(3)^2 = 40.5cm/s$

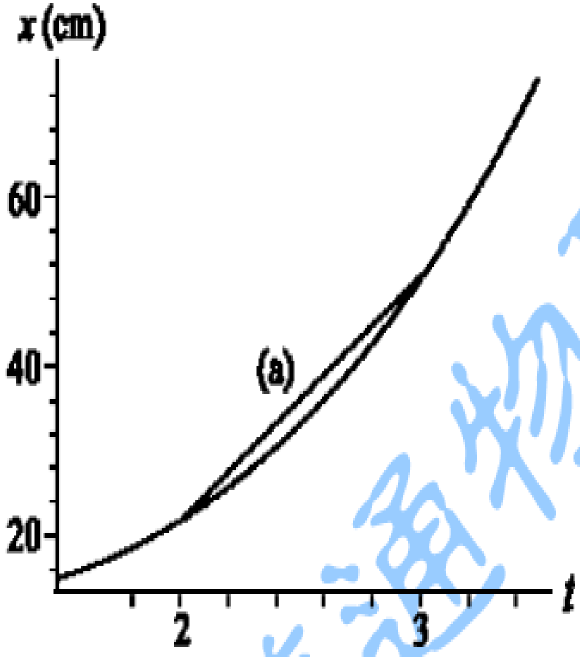
(d) $v(t = 2.5s) = 4.5(2.5)^2 = 28.13cm/s$

(e) $x_m = \dfrac{x(t = 2) + x(t = 3)}{2} = \dfrac{[9.75 + 1.5(2^3)] + [9.75 + 1.5(3^3)]}{2} = 36$

$t_m = \sqrt[3]{\dfrac{x_m - 9.75}{1.5}} \approx 2.596 \approx 2.6s$

$v(t = 2.6s) = 4.5(2.6)^2 = 30.42cm/s$

(f) 畫圖



8th Ed【Problem 2-19】: 9th Ed【Problem 2-19】

At a certain time a particle had a speed of 18m/s in the positive x direction, and 2.4s later its speed was 30m/s in the opposite direction. What is the average acceleration of the particle during this 2.4s interval?

某一時間，一質點以 18m/s 速度往 x 方向前進，2.4s 後，速度變成 30m/s 反方向，問 2.4s 間的平均加速度？

<解>： $a_{av}=\dfrac{(-30m/s)-(18m/s)}{2.4s}=-20m/s^2$

8^{th} Ed【Problem 2-21】: 9^{th} Ed【Problem 2-22】★

The position of a particle moving along the x axis depends on the time according to the equation $x = ct^2 - bt^3$, where x is in meters and t in seconds. What are the units of (a) constant c and (b) constant b? Let their numerical values be 3 and 2, respectively. (c) At what time does the particle reach its maximum positive x position? From t=0s to t=4s, (d) what distance does the particle move and (e) what is its displacement? Find its velocity at times (f) 1s, (g) 2s, (h) 3s, and (i) 4s. Find its acceleration at times (j) 1s, (k) 2s, (l) 3s, and (m) 4s.

一個粒子沿著 x 軸運動，其位置由方程式 $x = ct^2 - bt^3$ 決定，x 單位爲米，t 單位爲秒。（a）常數 c 的單位爲何？（b）常數 b 的單位爲何？令其常數分別是 3 和 2，（c）從 0-4 秒，在何時 x 有極大值？（d）粒子移動多少距離？（e）位移爲何？問當時間爲（f）1s（g）2s（h）3s（i）4s 時的速度？問當時間爲（j）1s（k）2s（l）3s（m）4s 時的加速度？

<解>：已知 $c = 3$ ， $b = 2$

(a) c 的單位： m/s^2

(b) b 的單位： m/s^3

(c) $v = \dfrac{dx}{dt} = 2ct - 3bt^2$

$t = 0$ 時， $v = 2ct - 3bt^2 = 0$

所以 $t = \dfrac{2c}{3b} = \dfrac{2(3m/s^2)}{3(2m/s^3)} = 1s$

(d) $x(1s) = 3(1)^2 - 2(1)^3 = 1$

$x(2s) = 3(2)^2 - 2(2)^3 = -4$

$x(3s) = 3(3)^2 - 2(3)^3 = -27$

$x(4s) = 3(4)^2 - 2(4)^3 = -80$

所以 0s-4s 從原點開始算，總共走的距離爲 82m。（0s-1s，走了 1m。1s-4s，走了 81m。）

(e) $x(0s) = 3(0)^2 - 2(0)^3 = 0$

所以位移爲 80m

(f) $v = \dfrac{dx}{dt} = 6t - 6t^2 m/s$

$v(1s) = (6)(1) - (6)(1)^2 = 0$

(g) $v(2s) = (6)(2) - (6)(2)^2 = -12$

(h) $v(3s) = (6)(3) - (6)(3)^2 = -36$

(i) $v(4s) = (6)(4) - (6)(4)^2 = -72m/s$

(j) $a = \dfrac{dv}{dt} = 2c - 6bt = 6 - 12tm/s^2$

$a(1s) = 6 - 12(1) = -6$

(k) $a(2s) = 6 - 12(2) = -18$

(l) $a(3s) = 6 - 12(3) = -30$

(m) $a(4s) = 6 - 12(4) = -42$

8^{th} Ed【Problem 2-25】: 9^{th} Ed【Problem 2-31】

Suppose a rocket ship in deep space moves with constant acceleration equal to $9.8 m/s^2$,which gives the illusion of normal gravity during the flight. (a) If it starts from rest, how long will it take to acquire a speed one-tenth that of light, which travels to $3\times10^8 m/s$? (b) how far will it travel in so doing?

假設一個太空船在太空深處以$9.8m/s^2$加速飛行。(a)如果它從靜止出發，多久才能達到光速的十分之一？(b)在此期間它走了多遠距離？

<解>：(a) $v = at$

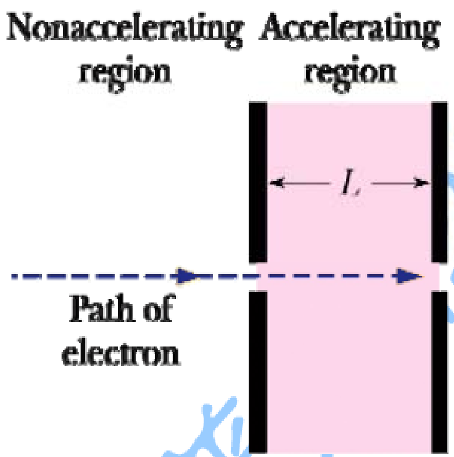
$$t=\frac{v}{a}=\frac{3\times10^7 m/s}{9.8m/s^2}=3.1\times10^6 s=35.4d$$

(b) $$x=\frac{1}{2}at^2=\frac{1}{2}(9.8m/s^2)(\frac{3\times10^7 m/s}{9.8m/s^2})^2=4.59\times10^{13} m$$

8th Ed【Problem 2-27】：9th Ed【Problem 2-23】

An electron with an initial velocity $v_0 = 1.5\times10^5 m/s$ enters a region of length L=1.00 cm where it is electrically accelerated. It emerges with $v_0 = 5.7\times10^6 m/s$. What is its acceleration, assume constant?

一電子以初速$v_0 = 1.5\times10^5 m/s$進入長一公分的加速區，以$v_0 = 5.7\times10^6 m/s$速度射出，其間爲等加速度，問等加速度爲何？



(圖 2-24)

<解>：$v^2 = v_0^2 + 2as$

$$a = \frac{v^2 - v_0^2}{2s} = \frac{(5.7\times10^6 m/s)^2 - (1.5\times10^5 m/s)^2}{2(0.01m)} = 1.6\times10^{15} m/s^2$$

8^{th} Ed【Problem 2-40】: 9^{th} Ed【Problem 2-34】

In Fig. 2-28, a red car and a green car, identical except for the color, moving toward each other in adjacent lanes(相鄰車道) and parallel to an x axis. At time $t=0$, the red car is at $x_r = 0$ and the green car is at $x_g = 220m$. If the red car has a constant velocity of 20km/h, the cars pass each other at $x = 44.5m$, and if it has a constant velocity of 40km/h, they pass each other at $x = 76.6m$. what are (a)the initial velocity and (b) the acceleration of the green car?

圖 2-28 中，一輛紅色車子及一輛綠色車子，面對面的行駛在相鄰車道，平行於 x 軸。在 $t=0$ 時，紅色車子位置在 $x_r = 0$，綠色車子位置在 $x_g = 220m$。如果紅色車子速度固定在 20 公里/小時，車子相遇時在 $x = 44.5m$。如果它固定速度為 40 公里/小時，車子相遇時在 $x = 76.6m$。問綠車的（a）初始速度和（b）加速度？

（圖 2-28）

<解>：1、紅車以$20km/h$固定速度前進，當與綠車相遇時，花了$\dfrac{44.5m}{20km/h}=\dfrac{44.5\times10^{-3}km}{20km/h}=8.01s$

綠車以$v_g(m/s)$初速度，$a_g(m/s^2)$加速度前進，走了$220-44.5=175.5m$

$$s=v_0t+\frac{1}{2}at^2$$

$$175.5=v_g(8.01)+\frac{1}{2}a_g(8.01)^2\text{..................1 式}$$

2、紅車以$40km/h$固定速度前進，當與綠車相遇時，花了$\dfrac{76.6m}{40km/h}=\dfrac{76.6\times10^{-3}km}{40km/h}=6.894s$

綠車以$v_g(m/s)$初速度，$a_g(m/s^2)$加速度前進，走了$220-76.6=143.4m$

$$s=v_0t+\frac{1}{2}at^2$$

$$143.4=v_g(6.894)+\frac{1}{2}a_g(6.894)^2\text{..................2 式}$$

（1式）和（2式）解方程式，得

$v_g=13.9m/s$

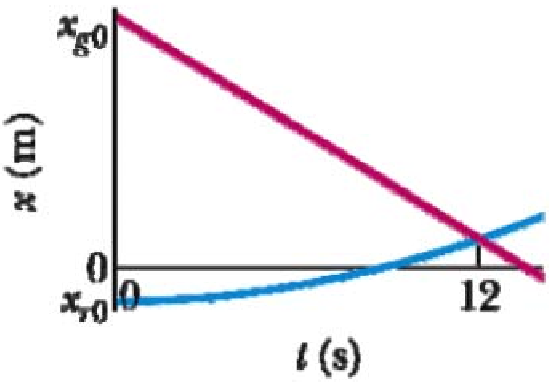
$a_g=2m/s^2$

8^{th} Ed【Problem 2-41】: 9^{th} Ed【Problem 2-35】

Figure 2-28 shows a res car and a green car that move toward each other. Figure 2-29 is a graph of their motion, showing the positions $x_{g0} = 270m$ and $x_{r0} = -35m$ at time $t = 0$. The green car has a constant speed of 20m/s and the red car begins from rest. What is the acceleration magnitude of the red car?

圖 2-28 顯示了一個紅車及綠車，相向移動。圖 2-29 是運動圖，顯示了在 $t = 0$ 時位置為 $x_{g0} = 270m$ 和 $x_{r0} = -35m$。綠車有固定速率為 20m/s，而紅車從靜止狀態開始運動。紅車加速度為何？

（圖 2-28）



（圖 2-29）

<解>：兩車在 12 秒時相遇

綠車走了 $20m/s \times 12s = 240m$

紅車走了 $270 - (-35) - 240 = 65m$

$$a = \frac{2s}{t^2} = \frac{2(65m)}{12^2} = 0.9m/s^2$$

8^{th} Ed【Problem 2-47】: 9^{th} Ed【Problem 2-45】

(a) With what speed must a ball be thrown vertically from ground level to rise to a maximum height of 50m? (b) How long will it be in the air? (c) Sketch graphs of y, v, and a versus t for the ball. On the first two graphs, indicate the time at which 50m is reached.

(a)問要用多快速度，才可以將一顆球，從地面垂直向上拋到最大高度 50 米的地方？ (b)這顆球在空氣中停留多久？ (c)畫出 y，v 和 a 對 t 圖。由 y-t 圖和 v-t 圖，指出對大高度 50 米的地方爲何？

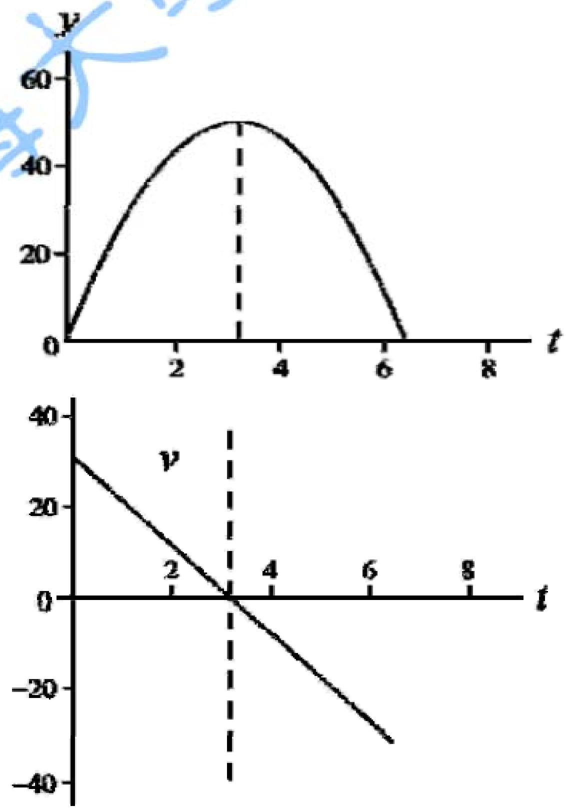
<解>：(a) $v^2 = v_0^2 + 2as$

$$v_0 = \sqrt{2gs} = \sqrt{2(9.8m/s^2)(50m)} = 31.3m/s$$

(b) $s = v_0 t - \dfrac{1}{2}gt^2$

$$t = \frac{2v_0}{g} = \frac{2(31.3)}{9.8} = 6.4s$$

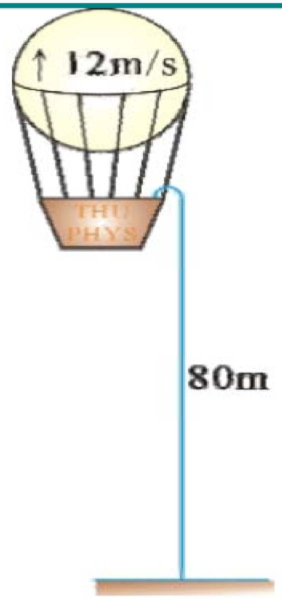
(c) 畫圖



8th Ed【Problem 2-49】: 9th Ed【Problem 2-49】

A hot-air balloon is ascending at the rate of 12m/s and is 80m above the ground when a package is dropped over the side. (a) How long does the package take to reach the ground? (b) With what speed does it hit the ground?

一熱氣球以 12 米/秒速率上升，在離地面 80 米時，一包裹被往下丟。(a)包裹著地要花多少時間？(b)落地速率爲何？



<解>：(a) $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$$\frac{1}{2}(9.8)t^2 - (12)t - (80) = 0$$

$$t = \frac{12 + \sqrt{12^2 - 4(\frac{9.8}{2})(-80)}}{2(\frac{9.8}{2})} = 5.4s$$

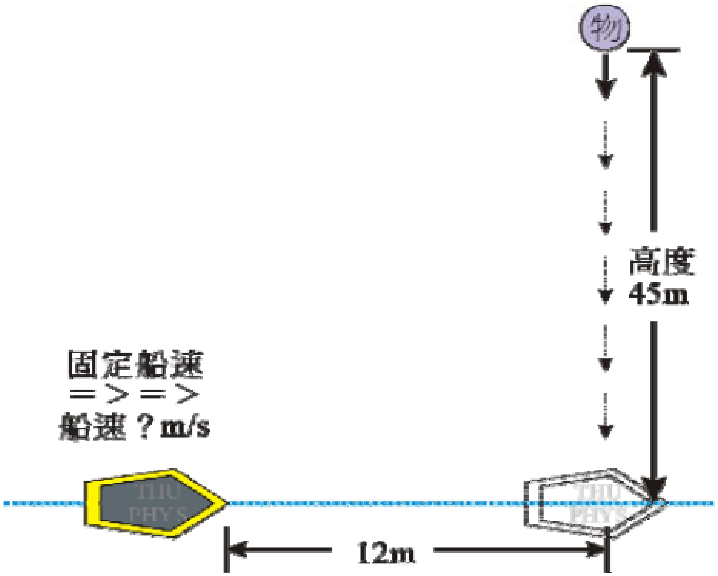
(b) $v = v_0 - gt = 12 - (9.8)(5.4) = -41 m/s$

著地時速率 $41 m/s$

8^{th} Ed【Problem 2-51】: 9^{th} Ed【Problem 2-53】

A key falls from a bridge that is 45m about the water. It falls directly into a model boat, moving with constant velocity, that is 12m form the point of impact when the key is released. What is the speed of the boat?

一個鑰匙從離水面 45 米的橋上落下。它直接掉落到一個速度固定的模型船上。當鑰匙一開始落下時，模型船離鑰匙掉落點 12 米。問船速爲何？



<解>： $45 = \frac{1}{2}(9.8)(t^2)$

鑰匙落到模型船所花的時間 $t = \sqrt{\frac{2(45m)}{9.8m/s^2}} = 3s$

模型船速度 $v_0 = \frac{12}{3} = 4m/s$

8^{th} Ed【Problem 2-57】: 9^{th} Ed【Problem 2-57】

To test the quality of a tennis ball, you drop it onto the floor from a height of 4m. It rebounds to a height of 2m If the ball is in contact with the floor for 12ms. (a) what is the magnitude of its average acceleration during that contact and (b) is the average acceleration up or down?

爲了檢驗網球的品質，你將它從 4 米高度丟到地板上，它回彈到 2 米的高度，如果球接觸地面時間爲 12 毫秒。(a) 碰撞時的平均加速度大小 (b) 平均加速度是往上還是往下？

<解>：(a) $v_1 = \sqrt{2gs} = \sqrt{2(9.8)(4)} = 8.85m/s$

$$v_2 = \sqrt{2g(\Delta s)} = \sqrt{2(9.8)(2)} = 6.26m/s$$

$$a_{av} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26m/s - (-8.85m/s)}{12\times 10^{-3}s} = 1.26\times 10^3 m/s^2$$

(b) 往上(因吾人訂定向上爲正)

8th Ed【Problem 2-63】: 9th Ed【Problem 2-61】

A steel ball is dropped from a building's roof and passes a window, taking 0.125s to fall from the top to the bottom of the window, a distance of 1.2m. It then falls to a sidewalk and bounces back past the window, moving from bottom to top in 0.125s. Assume that the upward flight is an exact reverse of the fall. The time the ball spends below the bottom of the window is 2s. How tall is the building?

一鋼球從一建築物的屋頂掉落穿過一窗戶，花了 0.125s 從窗口頂端掉到窗戶底部，距離 1.2 米。然後掉到人行道上，再反彈到窗口，從底部到頂端花了 0.125s。假設往上彈與往下掉的路徑一樣。球在窗戶下方花的時間為 2 秒。建築物有多高？

<解>：自由落下，所以鋼球 $v_0 = 0$

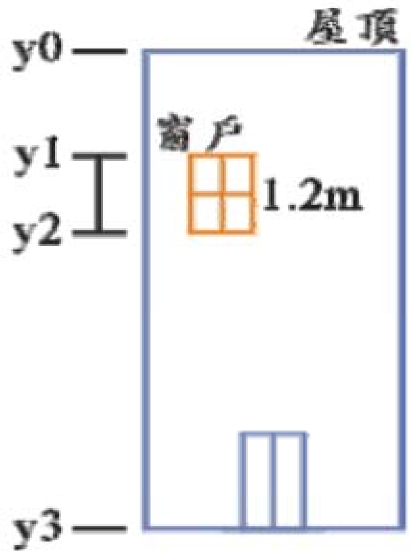
$$y_2 - y_1 = 1.2m$$

$$t_2 - t_1 = 0.125s$$

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}g(t_2 - t_1)^2$$

$$v_1 = \frac{1.2 - \frac{1}{2}(9.8)(0.125)^2}{0.125} = 8.99m/s$$

$$v_1^2 = 2gy_1 \Rightarrow y_1 = \frac{(8.99)^2}{2(9.8)} = 4.12m$$



在窗戶下方花的時間為 2 秒，因為球是來回，單程花的時間為 1 秒。

$$t_3 - t_1 = 1 + 0.125 = 1.125s$$

$$y_3 - y_1 = v_1(t_3 - t_1) + \frac{1}{2}g(t_3 - t_1)^2$$

$$y_3 - 4.12 = (8.99)(1.125) + \frac{1}{2}(9.8)(1.125)^2$$

$$y_3 = 20.4m$$

建築物高度 20.4 公尺。