GPN1-L08

Problem 1

Answer the following questions: (04小題)

$g = \frac{GM}{R^2} = \frac{0.0558}{(0.381)^2}\frac{GM_E}{R_E^2}$

$= 0.381 \times 9.8 = 3.767$

(a)Venus has a mass of about 0.0558 times the mass of Earth and a diameter of about 0.381 times the diameter of Earth. The acceleration of a body falling near the surface of Venus=_____ m/s^2

01: **ANS:=3.8**

(b)The mass of a planet is 1/100 that of Earth and its radius is 1/4 that of Earth. If a person weighs 600N on Earth, what would he weigh on this planet? W=_____ N

$g = \frac{\frac{1}{100}}{(\frac{1}{4})^2} \times 9.8 = 1.568 \quad W = mg = \frac{600}{9.8} \times 1.568 = 96$

02: **ANS:=96**

(c) An object at the surface of Earth (at a distance R from the center of Earth) weighs 90N. Its weight at a distance 3R from the center of Earth =_____ N

for $r > R \Rightarrow F \propto \frac{1}{r^2}$ $\quad 90 \cdot \frac{1}{3^2} = 10$

03: **ANS:=10**

(d) The approximate value of g at an altitude above Earth equal to one Earth diameter=_____ m/s^2

04: **ANS:=1.1**

$r = R + D = R + 2R = 3R$

$g \propto \frac{1}{r^2} \Rightarrow g = \frac{9.8}{3^2} = 1.089$

Problem 2

A spherical shell has inner radius R1, outer radius R2, and mass M, distributed uniformly throughout the shell. Find the magnitude F of the gravitational force exerted on the shell by a point mass particle of m, located a distance d from the center for the following 3 cases. (03小題)

(a)$d < R1$, F=_____ [M,R1,R2,G,d]

05: **ANS:=0**

(b)$R1 < d < R2$, F=_____ [M,R1,R2,G,d]

06: **ANS:=G*M*m*(d**3-R1**3)/(R2**3-R1**3)*(1/d**2)**

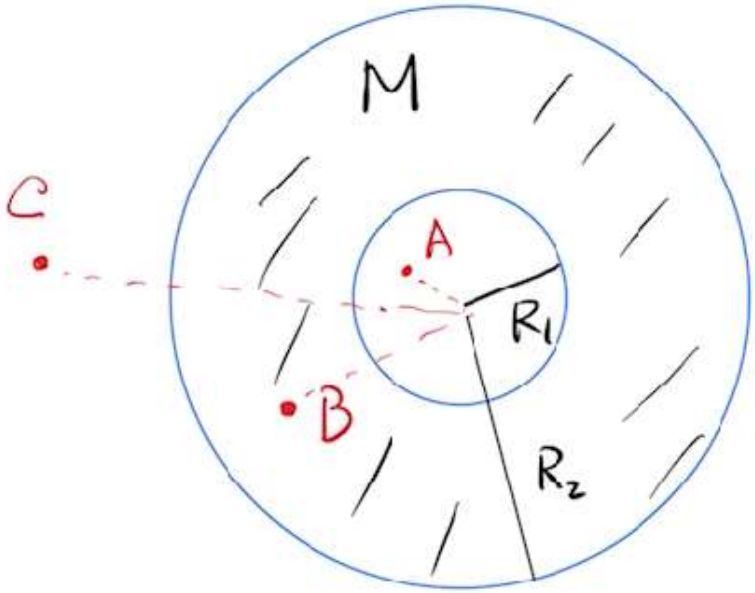
(c)$d > R2$, F=_____ [M,R1,R2,G,d]

07: **ANS:=G*M*m/d**2**

殼層定理(Shell Theorem)

1.球對稱物體對於球體外的重力貢獻如同將球體質量集中於球心。

2.在對稱球體內部的物體不受其外部球殼的重力影響。



A: $d<R_1 \Rightarrow g=0$, $F=0$

B: $R_1<d<R_2 \Rightarrow M'=\alpha M$, M'= mass inside d

$$F=\frac{GM'm}{r^2} \qquad \alpha=\frac{d^3-R_1^3}{R_2^3-R_1^3}$$

$$=\frac{G\left(\frac{d^3-R_1^3}{R_2^3-R_1^3}\right)Mm}{d^2}$$

$$=\frac{GMm}{d^2}\,\frac{d^3-R_1^3}{R_2^3-R_1^3}$$

C: $d>R_2 \Rightarrow F=\frac{GMm}{d^2}$

GPN1-L08/P02

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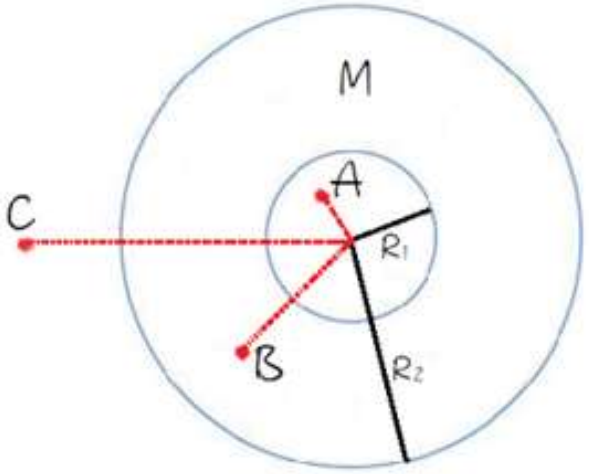


B點處的重力場是由下圖中綠色區域的質量所貢獻，外圍的黃色區域沒有貢獻(shell theorem)。所以我們要先算出綠色區的質量(M')

依據質量的体積比，我們設 $M' = \alpha M$

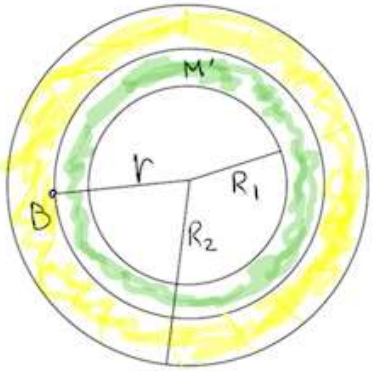
M = 整个殼層的總質量(黃+綠)

$M' < M,\ \alpha < 1$　　$M' \to V'$　　$M \to V$

$$V' = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3$$

$$V = \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3$$

$$\frac{V'}{V} = \frac{r^3 - R_1^3}{R_2^3 - R_1^3} = \frac{M'}{M} = \alpha$$



教授不好意思 看完您的影片後 我還是有點不懂 PR06的解是如何求出的 能否請您更詳細說明呢?

R1**3)*(1/d**2)

為甚麼需要加一個α呢

有點看不懂

B: $R < d < R_2 \Rightarrow M' = \alpha M$ (M'= mass inside d)

殼層定理(Shell Theorem)是古典重力學上的理論，其可簡化重力於對稱球體內部和外部的貢獻，並且在天文學上有特別的應用。殼層定理最先由牛頓在所推演出來，其闡明了

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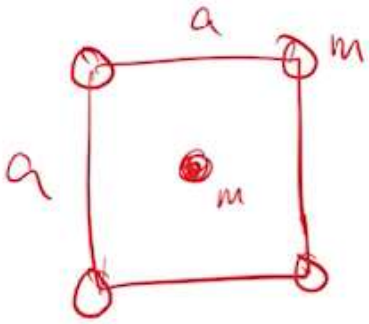
Problem 3

Answer the following questions: (03小題)

(a)Each of the four corners of a square with edge a is occupied by a point mass m. There is a fifth mass, also m, at the center of the square. To remove the mass from the center to a point far away, find the work that must be done by an external agent.
the work=_____ [G,m,a]

08: **ANS:=4*sqrt(2)*G*m**2/a**



$$W = \Delta U_g$$

$$= U_g(\infty) - U_g(0)$$

$$= 0 - \left(-\frac{Gm^2}{\frac{a}{\sqrt{2}}} \times 4\right)$$

$$= \frac{4\sqrt{2}Gm^2}{a}$$

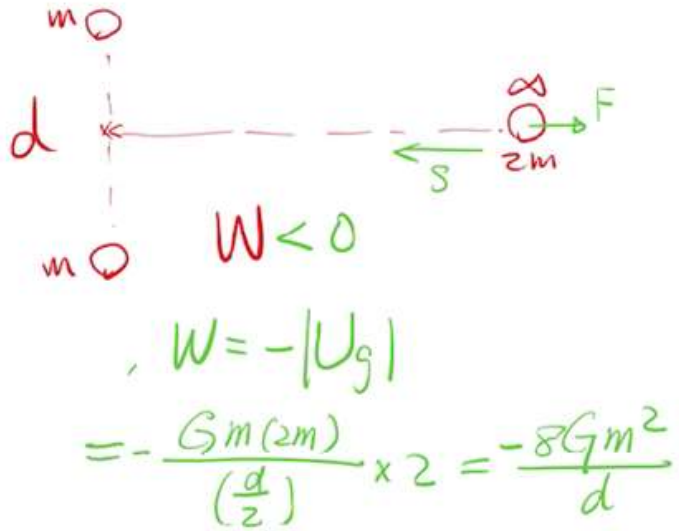
Problem 3

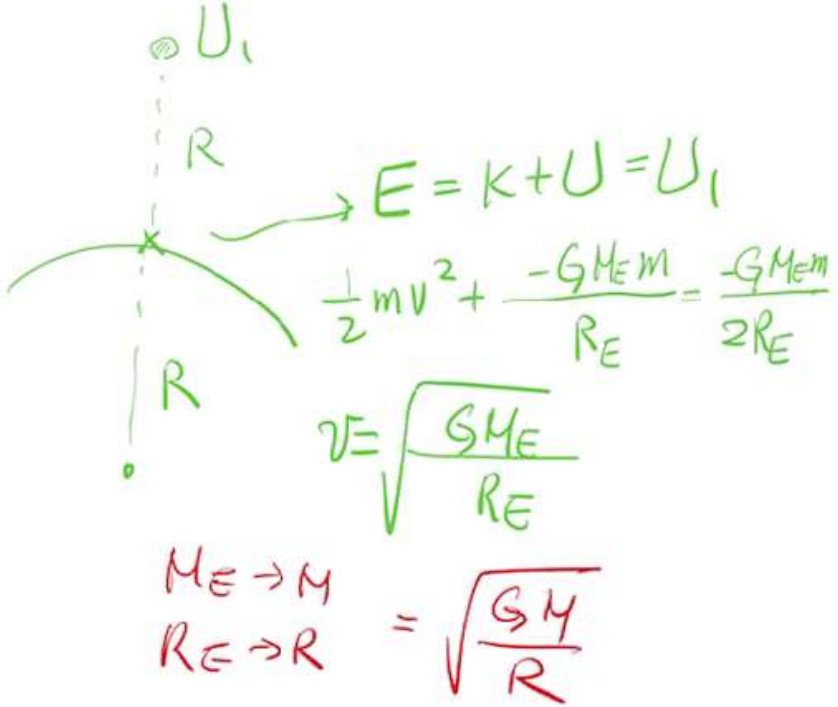
(b)Two particles, each of mass m, are a distance d apart. To bring a third particle, with mass 2m, from far away to a resting point midway between the two particles the work done by an external agent=_____ [G,m,d]

09: **ANS:=-8*G*m**2/d**2**

(c) An object is dropped from an altitude of one Earth radius above Earth's surface. If M is the mass of Earth and R is its radius the speed of the object just before it hits Earth is given by, v=_____ [G,M,R]

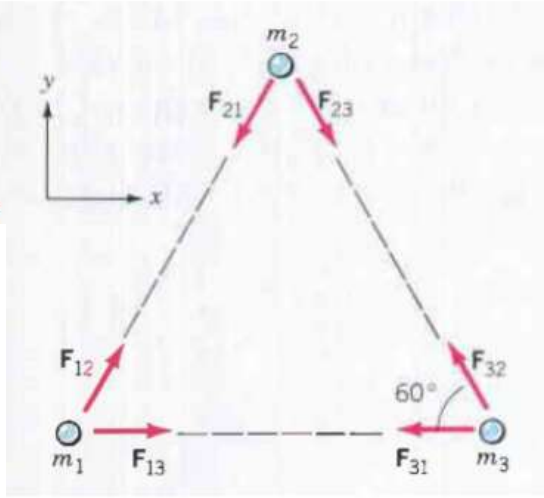
10: **ANS:=sqrt(G*M/R)**





Problem 4

Three point particles with masses $m_1 = 4$ kg, $m_2 = 2$ kg, and $m_3 = 3$ kg are at the corners of an equilateral triangle of side $L = 2$ m, as in the figure. Find the net force $\vec{F} = F_x\hat{i} + F_y\hat{j}$ on m_2. (02小題)

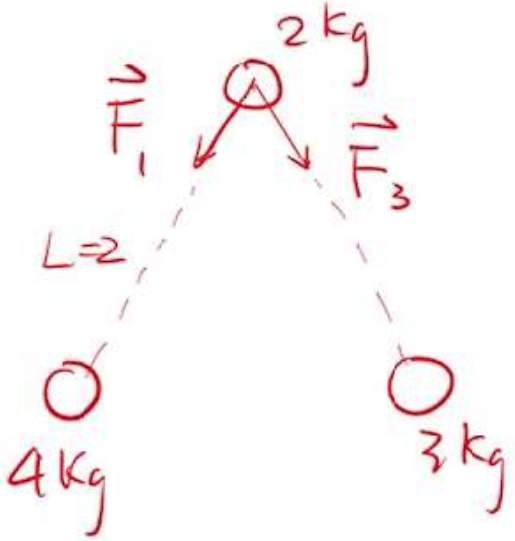


F_x=_____ N

11: **ANS:=-1.6E-11**

F_y=_____ N

12: **ANS:=-20.3E-11**



$$F_1 = \frac{G(4)(2)}{2^2} = \frac{8G}{4} = 2G$$

$$F_3 = \frac{6G}{2^2} = 1.5G$$

$$\vec{F}_1 = -\frac{1}{2}F_1\hat{i} - \frac{\sqrt{3}}{2}F_1\hat{j}$$

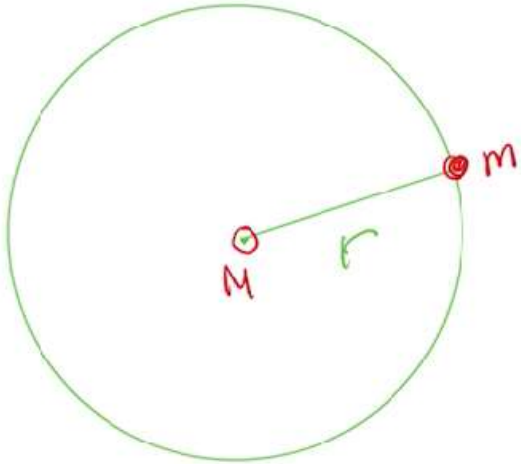
$$\vec{F}_3 = \frac{1}{2}F_3\hat{i} - \frac{\sqrt{3}}{2}F_3\hat{j}$$

$$\vec{F} = -0.5G\hat{i} - \frac{\sqrt{3}}{2}(3.5G)\hat{j}$$

$$= (-1.68, -20.22)$$

Problem 5

A satellite of mass m is in stable circular orbit of radius r. Find the following quantities: (a) the speed; (b) the period; (c) the linear momentum; (d) the kinetic energy; (e) the angular momentum? The mass of Earth is M. (05小題)



(a) the speed=_____ [G,m,M,r]

13: **ANS:=sqrt(G*M/r)**

(b) the period=_____ [G,M,m,r]

14: **ANS:=2*pi*sqrt(r**3/(G*M))**

(c) the linear momentum=_____ [G,m,M,r]

15: **ANS:=m*sqrt(G*M/r)**

(d) the kinetic energy=_____ [G,m,M,r]

16: **ANS:=(G*M*m)/(2*r)**

(e) the angular momentum=_____ [G,M,m,r]

17: **ANS:=m*sqrt(G*M*r)**

$$\frac{mv^2}{r} = \frac{GMm}{r^2}, \quad v = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$p = mv = m\sqrt{\frac{GM}{r}}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{GMm}{2r}$$

$$L = rp = m\sqrt{GMr}$$

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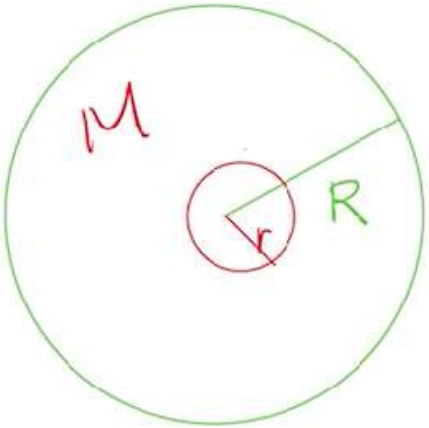
Problem 6

How does the field strength $g(r)$ vary inside a uniform solid sphere of density ρ=rho and radius R? (01小題)

$g(r)$=_____ [G,rho,R,r]

18: **ANS:=(4/3)*pi*G*rho*r**

殼層定理(Shell Theorem)
在對稱球體內部的物體不受其外部球殼的重力影響。



$$M(r)=\frac{r^3}{R^3}M \quad , \quad M=\frac{4}{3}\pi R^3\rho$$

$$g(r)=\frac{GM(r)}{r^2}=\frac{GM}{R^3}r$$

$$=\frac{4}{3}\pi\rho G r$$

Problem 6

Calculate the gravitational field strength at the surface of the following bodies (assumed to be uniform spheres): (a) Jupiter (1.9×10^{27} kg, radius 7.14×10^{7} m); (b) a neutron star of mass 10^{30} kg and radius 20 km. (02小題)

(a)g_{Jupiter}=_____ m/s^2

19: **ANS:=24.6**

(b)for neutron star, g=_____ m/s^2

20: **ANS:=1.67E11**

$$g=\frac{GM}{R^2}$$

$$g_{Jupiter}=\frac{6.67\times10^{-11}\times1.9\times10^{27}}{(7.14\times10^{7})^2}=24.9$$

$$g_N=\frac{6.67\times10^{-11}\times10^{30}}{(20\times10^{3})^2}=1.668\times10^{11}\ m/s^2$$

Problem 7

In its elliptical orbit, the speed of the earth at perihelion(遠日點) is $v_P = 3.03 \times 10^4$ m/s. If the distances to the sun at perihelion and aphelion(近日點) are $r_P = 1.47 \times 10^{11}$ m and $r_A = 1.52 \times 10^{11}$ m, find v_A. (01小題)

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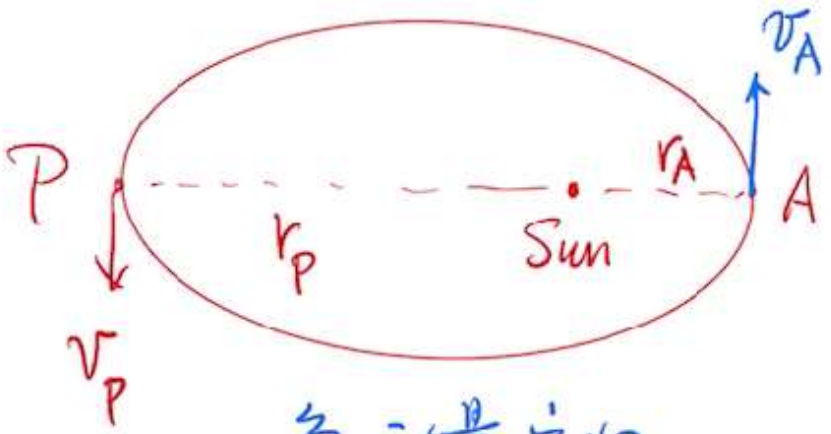
v_A=_____ m/s

21: **ANS:=2.93E4**

$L_P = L_A$

$r_P v_P = r_A v_A$



角動量守恆

$(1.47 \times 10^{11})(3.03 \times 10^4) = (1.52 \times 10^{11}) V_A$

$V_A = 2.93 \times 10^4$