GPN1-L09

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Problem 1

An object attached to one end of a spring makes 20 complete oscillations in 10 s. The mass of the object is 0.2 kg and the range of oscillation is from 10 cm to -10 cm with origin at the equillibrium point. Answer the following questions. (04小題)

(a)Its period=______ s

01: **ANS:=0.5**

(b)the angular frequency=______ rad/s

02: **ANS:=12.6**

(c)the spring constant k=______ N/m

03: **ANS:=31.6**

(d)the maximum speed=_____ m/s

04: **ANS:=1.257**

$$T = \frac{10}{20} = 0.5\ (s)$$

$$-10 \le X \le 10 \Rightarrow A = 10\ (cm) = 0.1\ m$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi = 12.57\ rad/s$$

$$\omega = \sqrt{\frac{k}{m}},\quad k = m\omega^2 = 0.2(12.57)^2 = 31.6\ N/m$$

$$V_{max} = \omega A = (12.57)(0.1) = 1.257\ m/s$$

Problem 2

A block attached to a spring oscillates in simple harmonic motion along the x axis. The limits of its motion are x = 10cm and x = 50 cm and it goes from one of these extremes to the other in 0.25 s. Find its amplitude and frequency. (02小題)

(a)amplitude=_____ m

05: **ANS:=<u>0.2</u>**

(b)frequency=______ Hz

06: **ANS:=<u>2</u>**

$$2A = (0.5 - 0.1) = 0.4$$

$$A = 0.2 \quad , \quad T = 0.25 \times 2 = 0.5$$

$$f = \frac{1}{T} = \frac{1}{0.5} = 2$$

Problem 3

A 3-kg block moves in simple harmonic motion according to $x = 2\cos(50t+3)$, where x is in meters and t is in seconds. Find its maximum velocity v_{max}. (03小題)

v_{max}=_______ m/s

07: **ANS:=100**

the period=____ s

08: **ANS:=0.126**

the spring constant=_____ N/m

09: **ANS:=7500**

$$v(t) = \frac{dx}{dt} = -2\sin(50t+3)(50)$$

$$= -100\sin(50t+3)$$

$\uparrow v_{max}$

$$50t+3 = \omega t+3, \quad \omega = 50 = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{50} = 0.126\ (s)$$

$$k = m\omega^2 = 3(50)^2 = 7500\ N/m$$

Problem 3

A 0.20-kg object attached to a spring whose spring constant is 500 N/m executes simple harmonic motion. If its maximum speed is 5.0 m/s, find the amplitude of its oscillation. (01小題)

amplitude=_____ m

10: **ANS:=0.1**

$$v_{max} = 5 = \omega A, \quad \omega = \sqrt{\frac{k}{m}}$$

$$A = \frac{5}{\omega} = 5\sqrt{\frac{m}{k}} = 5\sqrt{\frac{0.2}{500}} = 0.1$$

Problem 4

A particle is in simple harmonic motion along the x axis. The amplitude of the motion is x_m. When it is at $x = x_1$, its kinetic energy is $K = 5$ J and its potential energy (measured with $U = 0$ at $x = 0$) is $U = 3$ J. When it is at $x = -1/2x_1$, find the kinetic and potential energies. (02小題)

K=______ J

11: **ANS:=6**

U=______ J

12: **ANS:=2**

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx_1^2 = 5+3 = 8$$

$$= \frac{1}{2}kx_m^2 = 8$$

$$x = -\frac{1}{2}x_m \Rightarrow U' = \frac{1}{2}k\left(\frac{1}{4}x_m^2\right) = \frac{1}{4}\cdot 8 = 2$$

$$K' = E - U' = 8-2 = 6$$

Problem 5

Answer the following questions: (04小題)

$$E = 6 = \frac{1}{2}kA^2 = \frac{1}{2}(200)A^2$$

$$A = \frac{1}{10}\cdot\sqrt{6} = 0.245$$

(a)A 0.25-kg block oscillates on the end of the spring with a spring constant of 200 N/m. If the system has an energy of 6.0 J, then the amplitude of the oscillation=_____ m

13: **ANS:=0.24**

(b)A 0.25-kg block oscillates on the end of the spring with a spring constant of 200 N/m. If the oscillation is started by elongating the spring 0.15 m and giving the block a speed of 3.0 m/s, then the maximum speed of the block=_____ m/s

14: **ANS:=5.2**

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}(200)(0.15)^2 + \frac{1}{2}(0.25)(3)^2 = 3.375$$

$$\frac{1}{2}mv_x^2 = 3.375, \quad v_x = 5.20$$

(c)Following (b), the amplitude of the oscillation=_____ m

15: **ANS:=0.18** $\frac{1}{2}kA^2 = 3.375, \quad A = 0.184$

(d)A block attached to a spring undergoes simple harmonic motion on a horizontal frictionless surface. Its total energy is 50 J. When the displacement is half the amplitude, the kinetic energy=____ J

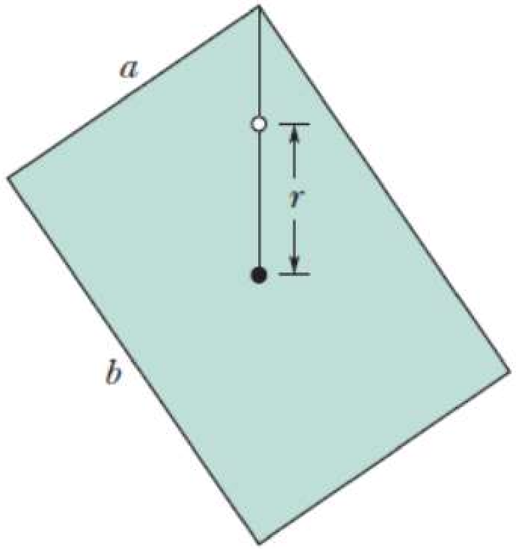
16: **ANS:=37.5**

$$E = 50 = \frac{1}{2}kA^2$$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{1}{4}(50)$$

$$K = E - U = 50 - \frac{50}{4} = 37.5$$

Problem 6



A rectangular block of mass m, with face lengths a and b , is to be suspended on a thin horizontal rod running through a narrow hole in the block. The block is then to be set swinging about the rod like a pendulum, through small angles so that it is in SHM. The figure shows one possible position of the hole, at distance r from the block's center, along a line connecting the center with a corner. (02小題)

(a)the period $T(r)$ of the pendulum versus distance r, $T(r)$=_____ [r,a,b,g]

17: **ANS:=(2*pi/g)*sqrt((a**2+b**2)/(12*r)+r)**

(b)For what value of r does that minimum of period occur? r=____ [a,b]

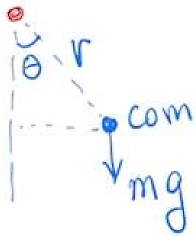
18: **ANS:=sqrt((a**2+b**2)/12)**

基本公式：轉动運动的方程式

$\tau = I\alpha$, $-mgr\sin\theta = I\alpha$

small angle approximation:

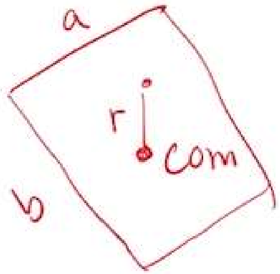
$\sin\theta \simeq \theta$



$\alpha = -\frac{mgr}{I}\theta$, $a = \frac{-k}{m}x = -\omega^2 x$

$\Rightarrow \omega = \sqrt{\frac{mgr}{I}}$ $\quad \omega = \sqrt{\frac{k}{m}}$

$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgr}}$



$I_{com} = \frac{1}{12}(a^2+b^2)m$

$I = \frac{1}{12}m(a^2+b^2) + mr^2$

$T = 2\pi\sqrt{\frac{\frac{1}{12}m(a^2+b^2)+mr^2}{mgr}}$

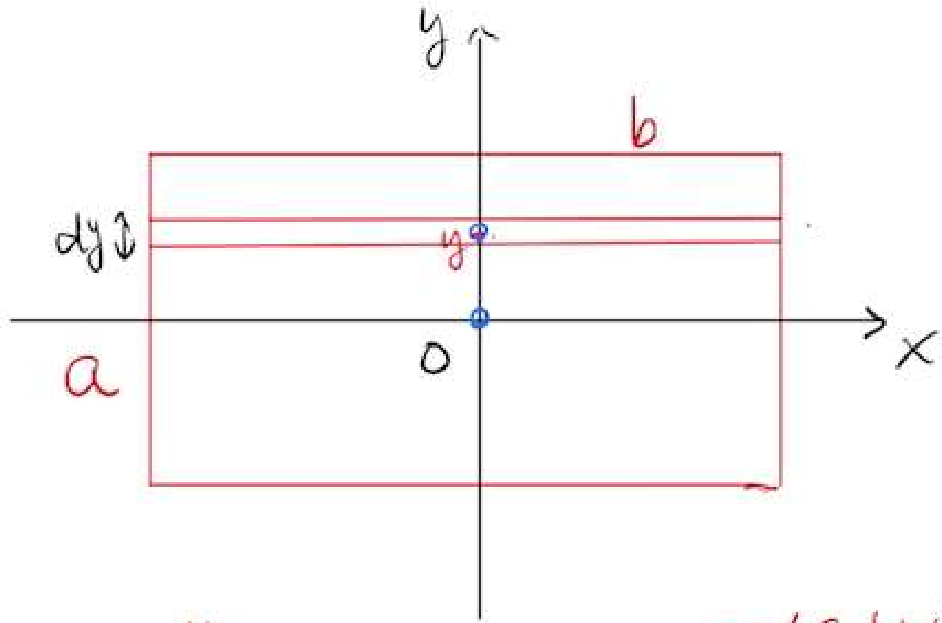
$= 2\pi\sqrt{\frac{\frac{a^2+b^2}{12r}+r}{g}}$

$\frac{dT}{dr} = 0 \Rightarrow \frac{d}{dr}\left(\frac{a^2+b^2}{12r}+r\right) = 0$

$-\frac{a^2+b^2}{12}r^{-2} + 1 = 0 \Rightarrow r = \sqrt{\frac{a^2+b^2}{12}}$

這個問題基本上是積分的應用，我們要求一個長方形的轉動慣量，基本上要進行二重積分，但是因為我們已經知道一個細長棒子相對於中心點的轉動慣量$=1/12Mb^2$，因此我們只需要對這些微量的細長棒子做一維積分就可以。每一個細長棒子的轉動慣量相對於中心點的轉動慣量固然是$1/12Mb^2$，但是相對於質心就必須使用平行軸定理將轉動軸細長棒子的中心點移動到整個長方形的質心，轉動軸平行移動的距離是y，利用平行軸定理我們可以寫下這個細棒子相對於質心的轉動慣量$dI=1/12\ dm\ b^2 + dm\ y^2$。最後再將所有的細長棒子求和，也就是dy積分，-a/2到a/2的積分。也就是兩倍的0到a/2的積分。

推導公式：$I=\frac{1}{12}m(a^2+b^2)$



$$dm=\frac{dy}{a}m=\frac{m}{a}dy$$

平行軸定理

平移y的距離

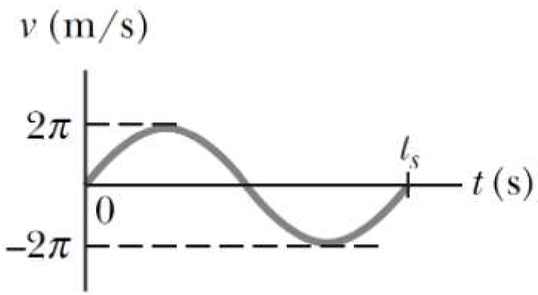
$$dI=\frac{1}{12}dm\,b^2+dm\,y^2$$

$$I=\int dI=2\times\int_0^{\frac{a}{2}}\frac{b^2}{12}\frac{m}{a}dy+\frac{m}{a}y^2dy$$

$$=2\left\{\frac{b^2m}{12a}\int_0^{\frac{a}{2}}dy+\frac{m}{a}\int_0^{\frac{a}{2}}y^2dy\right\}=\frac{2m}{a}\left\{\frac{b^2}{12}\cdot\frac{a}{2}+\frac{1}{3}\left(\frac{a}{2}\right)^3\right\}$$

$$=\frac{2m}{a}\left\{\frac{1}{24}ab^2+\frac{a^3}{24}\right\}=\frac{1}{12}m(a^2+b^2)$$

Problem 7



A simple harmonic oscillator consists of a block attached to a spring with $k = 200$ N/m. The block slides on a frictionless surface, with equilibrium point $x = 0$ and amplitude 0.20 m. A graph of the block's velocity v as a function of time t is shown in the figure. The horizontal scale is set by $t_s = 0.20$ s. What are (a) the period of the SHM, (b) the block's mass, (c) its displacement at $t = 0$, (d) its acceleration at $t = 0.10$ s, and (e) its maximum kinetic energy? (05小題)

$v(t) = 2\pi \sin\left(\frac{2\pi}{t_s} t\right)$

$= 2\pi \sin(10\pi t)$

(a) the period of the SHM=______ s

19: **ANS:=0.2**

$T = t_s = 0.2$, $\omega = 10\pi$

(b) the block's mass=______ kg

20: **ANS:=0.203**

$\omega = \sqrt{\frac{k}{m}} \Rightarrow 10\pi = \sqrt{\frac{200}{m}}$, $m = \frac{2}{\pi^2} = 0.203$

(c) its displacement at $t = 0$=______ m

21: **ANS:=-0.2**

$v_x = 2\pi = \omega A = 10\pi(0.2) = 2\pi$ (YES)

$t = 0, v = 0 \Rightarrow x(0) = A$ or $-A$

$v(0^+) > 0 \Rightarrow x(0) = -A = -0.2$

$x(t) = -A\cos\omega t = -0.2\cos(10\pi t)$

(d) its acceleration at $t = 0.10$ s=______ m/s^2

22: **ANS:=-197**

$a(t) = A\omega^2\cos\omega t = (0.2)(10\pi)^2\cos(10\pi t)$

$a(0.1) = (0.2)(10\pi)^2\cos(\pi) = -197.4$

(e) its maximum kinetic energy=______ J

23: **ANS:=4**

$K_x = \frac{1}{2} m v_x^2 = \frac{1}{2}(0.203)(2\pi)^2 = 4.007$