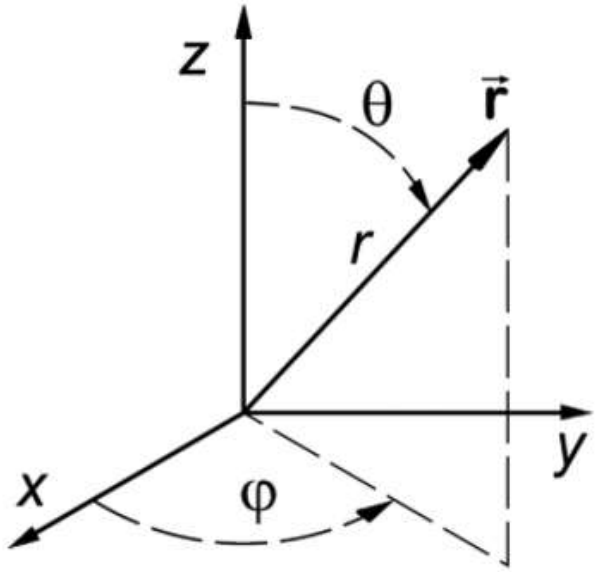
GPN1-MID

Problem 1

The spherical coordinatesr, θ, ϕ are defined as in the figure. The vector $\vec{r}$ is given by the sphereical coordinates: $r = 10, \theta = 40^\circ, \phi = 65^\circ$. We can also describe this vector in Cartisian coordinates $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Find the components of $\vec{r}$ in Cartisian coordinates. (05小題)



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

(a)The length of the vector of $\vec{r}$ projected(投影) on the xy-plane(xy-平面).

01: **ANS:=10*sin(pi*40/180)** =6.428

(b)x=____

02: **ANS:=2.717**

(c)z=____

03: **ANS:=7.66**

(d)Suppose we have a vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, find the angle θ and ϕ.
θ=_____ degree.

04: **ANS:=42.03**

(e)ϕ=_____ degree

05: **ANS:=56.3**

Problem 2

A particle is moving in a straight line. The position of the particle is given as a function of time : $x(t) = 6t^3 - 5t^2 + 7t - 3$. (a)Find the average velocity of the particle during the time interval from t=3 to 5. (b)What is the instantaneous velocity of the particle at t=4. (c)What is the instantaneous acceleration of the particle at t=4. (d)The mass of the particle is 5 kg. What is the average force exerting on the particle from t=3 to t=5? All quantities are in SI units. (04小題)

(a)v_{avg} ==_____ m.s

06: **ANS:=261**

(b)$v(4)$ ==_____ m.s

07: **ANS:=255**

(c)$a(4)$ ==_____ m.s

08: **ANS:=134**

(d)the average force, F_{avg} ==_____ N

09: **ANS:=670**

Problem 2

A particle is moving in a circle of radius r and constant speed v. Write the coordinates of this particle as a function of time t. The initial position of the particle is $x(0) = r,\ y(0) = 0$. (02小題)

$x(t)$=_____ [r,v,t]

10: **ANS:=r*cos(v/r*t)**

$y(t)$=_____ [r,v,t]

11: **ANS:=r*sin(v/r*t)**

Problem 3

打擊者擊打棒球，使其以 $\theta = 53°$ 的速度 $v_0 = 37.0$ m/s 飛離球棒。

PART 1:以打擊的初始位置為原點， 在 t=2.0 s時，球的 (a)位置及其 (b)速度。 求球到達飛行最高點的 (c) 時間t_1，以及此時的 (d) 高度 h。 (e) 計算水平射程 R。 (09小題)

(a)position at t=2 s:
x=_____ m

12: **ANS:=44.4**

y=_____ m

13: **ANS:=39.6**

(b)the speed at t=2 s, $|v|$=_____ m/s

14: **ANS:=24.35**

(c)t_1=_____ s

15: **ANS:=3.02**

(d)h=_____ m

16: **ANS:=44.7**

(e)R=_____ m

17: **ANS:=134**

PART.2 For (f) and (g), during the first 2 s, find:
(f)the work done by the gravitational force=_____ J

18: **ANS:=388**

(g)the impulse of the gravitational force=_____ kg.m/s

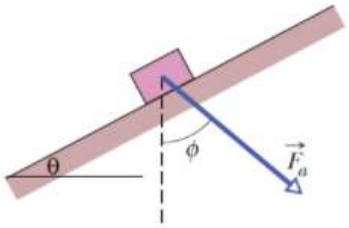
19: **ANS:=9.95**

PART 3.(h)if the ball explodes at t=2 into 2 pieces of equal masses. One takes a free fall from rest, find the x coordinate of the other piece when it lands on the ground. x=_____ m

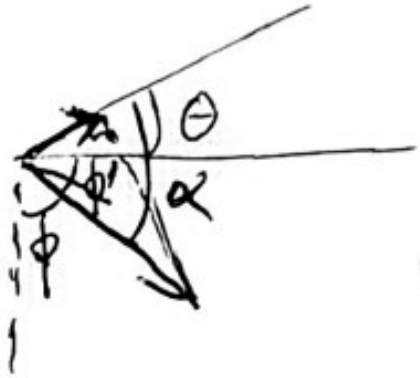
20: **ANS:=199.8**

Problem 4

在圖中，恆定力 $\vec{F}_a$ 以角度 $\phi = 53^0$ 施加到 3 kg 的盒子上，導致盒子以恆定速度向上移動無摩擦斜坡。 斜坡的角度為$\theta = 45°$。 (a) 當盒子移動了垂直距離 h=0.5 m 時，$\vec{F}_a$ 對盒子做了多少工作W_F？ (b) 求力F_a。 (c) 求正向力N|。 (03小題)



$\vec{F}_a$ = _____ N



$\alpha = \phi' + \theta$

$= 90° - 53° + 45°$

$= 82°$

21: ANS:=149.4

$W_F = mgh = 3 \times 9.8 \times 0.5 = 14.7$

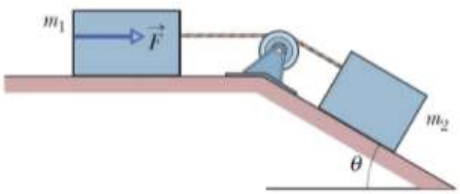
$F\cos\alpha = mg\sin\theta$

$F\cos 82° = 3(9.8)\sin 45°$

$F = 149.4$

$F\sin\alpha + mg\cos\theta = N = 68.73$

Problem 4



圖中顯示了一個質量為 $m_2 = 1.0\,\text{kg}$ 的盒子，位於傾斜角度 $\theta = 30°$ 的無摩擦斜面上。 它通過質量可以忽略不計的繩索連接到水平無摩擦表面上質量為 $m_1 = 3.0\,\text{kg}$ 的盒子。 滑輪是無摩擦和無質量的。 如果水平力F 的大小為 2.3 N，(a) 連接線的張力T是多少？ (b) 加速度a是多少？ (c) 在不鬆弛的情況下，F_{max} 的可能的最大值是多少？ (03小題)

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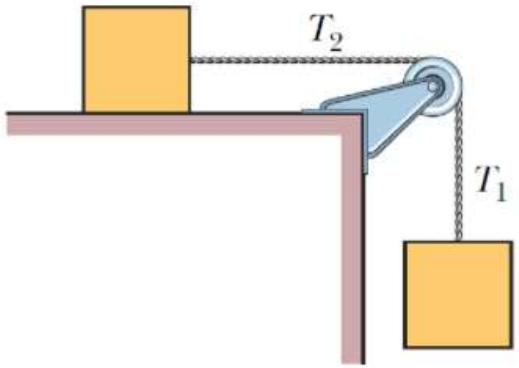
the acceleration a= _____ m/s^2

22: **ANS:=1.8**

Problem 5

Part.1 In the figure, two blocks of mass M are connected by a massless string over a masslss pulley. The string does not slip on the pulley and the pulley's axis is frictionless. The surface of table is smooth(frictionless). When this system is released from rest, the acceleration of the blocks is constant. What are (a)the magnitude of either block's acceleration a? (03小題)



a=____ [M,g]

23: **ANS:=1/2*g**

PART.2

Refer to the same figure, two blocks of mass M are connected by a massless string over a masslss pulley. The string does not slip on the pulley and the pulley's axis is frictionless. Coefficient of kinetic friction between table surface and the block is μ_k. When this system is released from rest, the acceleration of the blocks is constant. What is the magnitude of either block's acceleration a?

a=____ [M,g]

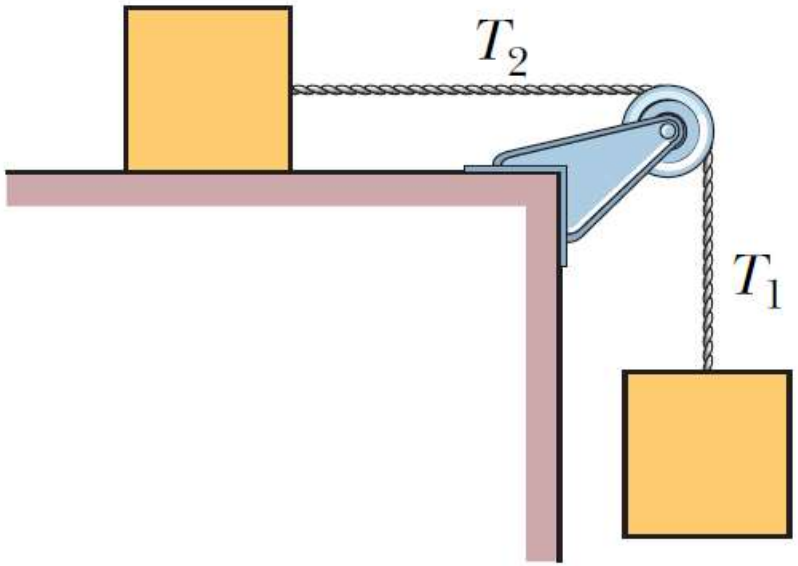
24: **ANS:=** $a = \frac{1-\mu_k}{2} g$

PART.3

Refer to the sam figure, two 6.20 kg blocks are connected by a massless string over a pulley of radius $R = 2.40$ cm and rotational inertia $I = 7.40 \times 10^{-4}$ kg.m^2. The string does not slip on the pulley; it is **not known** whether there is friction between the table and the sliding block; the pulley's axis is frictionless. When this system is released from rest, the pulley turns through 1.30 rad in 91.0 ms and the acceleration of the blocks is constant. What are (a)the magnitude of either block's acceleration a?

a=____ m/s^2

25: **ANS:=7.54**



$$\Delta\theta = \frac{1}{2}\alpha t^2$$

$$1.3 = \frac{1}{2}\alpha (0.091)^2$$

$$\alpha = 314 \text{ rad/s}^2$$

$$a = R\alpha = (0.024)(314) = 7.535$$

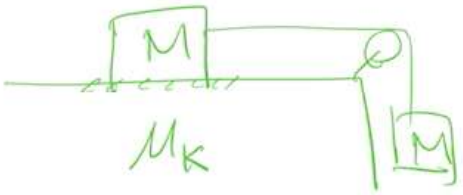
$$m_1 g - T_1 = m_1 a$$

$$T_1 = m_1(g-a) = 6.2(9.8-7.535) = 14.04$$

$$(T_1 - T_2)R = I\alpha$$

$$(14.04 - T_2)(0.124) = (7.4\times10^{-4})(314)$$

$$T_2 = 4.358$$



$$Mg - T = Ma$$

$$T = Ma$$

$$Mg = 2Ma$$

$$a = \frac{1}{2}g$$

$$T = Ma = M\left(\frac{1}{2}g\right) = \frac{1}{2}Mg$$

$$Mg - T = Ma$$

$$T - \mu_k Mg = Ma$$

$$Mg - Ma - \mu_k Mg = Ma$$

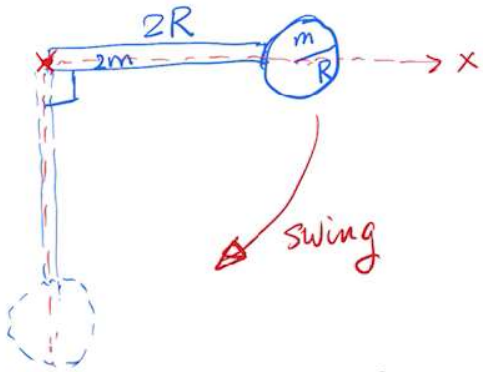
$$2a = (1-\mu_k)g, \quad a = \frac{1-\mu_k}{2}g$$

$$T = M(g-a) = Mg\left(1 - \frac{1-\mu_k}{2}\right) = Mg\left(\frac{1+\mu_k}{2}\right)$$

A force F=42.36N pushing m2 to the right.

Problem 6

請計算圖形中的剛體相對於轉動軸的轉動慣量。剛體中各物體的質量和長度如圖所示，長竿子和圓盤的質量都是均勻分佈，長竿子的質量為2m長度為2R，圓盤的質量為m半徑為R。假設現在這個剛體自靜止開始釋放，當這個剛體的中心軸線擺盪至鉛垂線時，此剛體的角速率，此剛體的轉動動能。 (03小題)



I=_____ kg.m^2

m=0.5 kg, R=1.2 m.

I=?, ω=?, K=?

26: **ANS:=8.76**

ω=_____ rad/s

27: **ANS:=2.59**

K=_____ J

28: **ANS:=29.4**

$$I=\frac{1}{3}(2m)(2R)^2+\frac{1}{2}mR^2+m(3R)^2$$

$$=\left(\frac{8}{3}+\frac{1}{2}+9\right)mR^2=\frac{73}{6}mR^2$$

I=12.16 mR^2

$$x_{com}=\frac{1}{3m}(2mR+m(3R))=\frac{5}{3}R$$

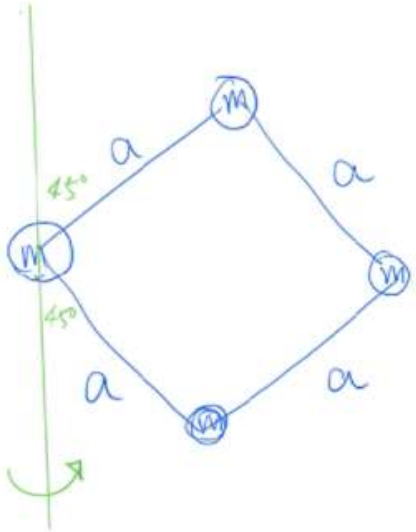
$$\Delta U=-(3m)g\left(\frac{5}{3}R\right)=-5mgR$$

$$\frac{1}{2}I\omega^2=-\Delta U,\ \frac{1}{2}\left(\frac{73}{6}mR^2\right)\omega^2=5mgR$$

$$\omega=\sqrt{\frac{60}{73}\frac{g}{R}}$$

Problem 6

4 particles of equal mass ($m = 0.2$ kg) are sitting on the corners of a square of side $a = 2.5$ m. The square is oriented as in the figure and rotates about the vertical axis as shown. The masses of the rods connecting the particles are negligible. Find the moment of inertia. (01 小題)



I=_____ kg.m^2

29: **ANS:=3.75**

$$m\left(\frac{a}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}a)^2$$

$$= ma^2 + 2ma^2 = 3ma^2$$

$$= 3(0.2)(2.5)^2 = 3.75$$

Problem 7

我們假設有一個恆星其質量是3.5×10^{30} kg，如果在距離這個恆星2×10^{11} m上測量單位面積上恆星的光照強度為4.8×10^{3} W/m^2，可以回推這個恆星的發光功率。 (04小題)

(a)以這個恆星為球心，計算以測量者所在處與恆星的距離為半徑之球面的面積=_____ m^2

30: **ANS:=4*pi*2.0E11**2** =5.027E23

(b)恆星的發光功率=_____ W

31: **ANS:=2.413E27**

PART.2(c)For a microwave of wave length 1 cm find its frequency=_____ Hz

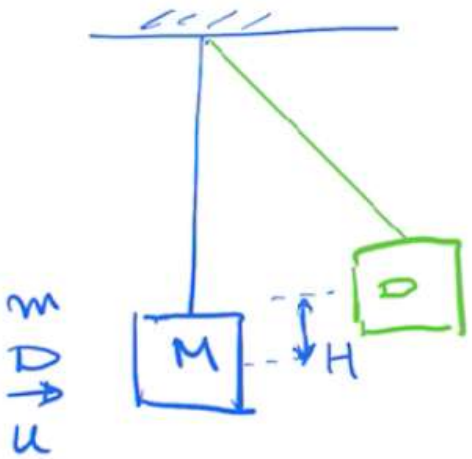
32: **ANS:=3E8/0.01**

PART.3(d)1 eV=_____ Joule

33: **ANS:=1.6E-19**

Problem 7

As shown in the figure, a bullet(子彈), of mass $m = 100$ g and speed u, is fired into a block of mass $M = 0.5$ kg which is suspended(懸掛) on the ceiling(天花板). The bullet embeds(沒入) in the block and raises(提高) it by a height $H = 20$ cm. Find the speed of the bullet u? (02小題)



(a)u=_____ m/s

38: **ANS:=11.87**

(b)the ratio of the thermal energy to the initial kinetic energy of the bullet=_____

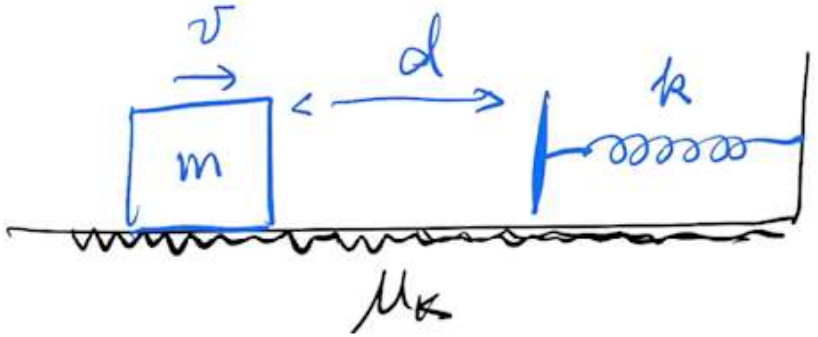
39: **ANS:=0.8333**

Problem 8

A 20 kg block is 3.5 m away from a stiff spring and moving towards it at 5.0 m/s along a rough horizontal surface. The spring has an elastic constant k = 500 N/m, and it gets compressed 0.50 m after the block strikes it. What is the coefficient of kinetic friction between the block and the rough surface? (01小題)

coefficient of kinetic friction=_____

38: **ANS:=0.239**



$$\Delta E = \Delta K + \Delta U$$

$$= \frac{1}{2}(20)(5)^2 + \frac{1}{2}(500)(0.5)^2$$

$$= -187.5 = W_{nc} = -f_r \cdot (3.5+0.5)$$

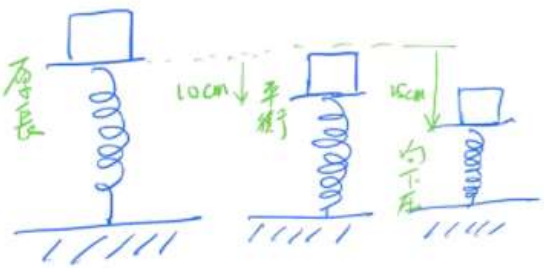
$$f_r = 46.875 = \mu_K (20)(9.8)$$

$$\mu_K = 0.239$$

Use 3.5 instead of 3.5+0.5
Accept 0.273

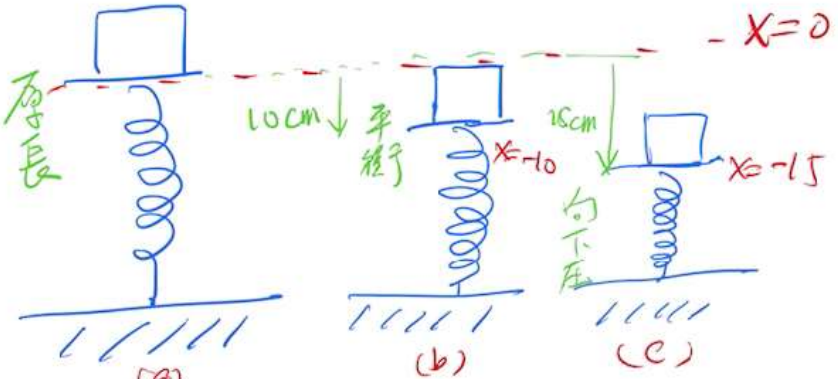
Problem 8

一個鐵塊被約束在一個無摩擦的垂直軸中移動。 如圖所示，鐵塊連接到一個輕型垂直彈簧。 在平衡時，彈簧被壓縮 10 cm。 從該配置開始，將鐵塊進一步向下壓，使彈簧總壓縮量變為 15 cm。 然後釋放鐵塊。 假設鐵塊仍然連接在彈簧上，那麼在釋放後，鐵塊會上升多少距離？ (01小題)



鐵塊會上升的距離=_____ cm

39: **ANS:=10**



15 cm 為單位　　以 x=-15 為重力位能的零點

(b) $\Rightarrow mg = k(10)$　$\frac{mg}{k} = 10$

(c) $\Rightarrow U_i = \frac{1}{2}k(15)^2 - 0$

設彈射最高點 = x

$U_f = \frac{1}{2}kx^2 + mg(x+15)$

$U_i = U_f$, $\frac{1}{2}k(15^2) = \frac{1}{2}kx^2 + mg(x+15)$

$15^2 = x^2 + \frac{2mg}{k}(x+15)$

$225 = x^2 + 20x + 300$

$x^2 + 20x + 75 = 0$

$x = -5, -15$

$\Rightarrow h = -5 - (-15) = 10$ (cm)