

Problem 1

GPN1-Q01

Gold, which has a density of 19.32 g/cm^3 , is the most ductile(韌性的) metal and can be pressed into a thin leaf or drawn out into a long fiber. (a) If a sample of gold, with a mass of 38.5 g , is pressed into a leaf of $2.0 \mu\text{m}$ thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius $1.5 \mu\text{m}$, what is the length of the fiber? (02小題)

(a) the area = _____ m^2

03: ANS:=0.9964

(b) the length = _____ m

04: ANS:=2.819E5

$$N = \frac{100}{107.9} \times 6.02 \times 10^{23} = 5.579 \times 10^{23}$$

$$V = \frac{M}{D} = \frac{100}{10.5} = 9.52 \text{ cm}^3$$

$= 9.52 \times 10^{-6} \text{ m}^3$



$$= AL = \pi \times (10^{-3})^2 L, \quad L = 3.03 \text{ m}$$

$$= At = \ell^2 (10^{-3})$$

$\ell = 9.76 \text{ cm}$
 $= 0.0976 \text{ m}$

```
#gold-----  
D=19.32 #g/cm^3  
A=197 #gold  
M=38.5 #g  
r=1.5E-3 #mm  
t=2E-3 #mm  
NA=6.02E23  
N=M/A*NA  
V=(M/1000)/(D*1000)  
L=V/(pi*(r/1000)**2)  
Area=V/(t/1000)  
print('N= {0:.3e}'.format(N))  
print('Area= {0:.3e}'.format(Area))  
print('L= {0:.3e}'.format(L))
```

```
N= 1.176e+23  
Area= 9.964e-01  
L= 2.819e+05  
d= 9.982e-01  
Q= 1.882e+04
```

Problem 1

The rotation of a planet rotates once every 15 hours. Please calculate the angle that the rotation axis of the earth rotates for 1 minute. (01小題)

the angle=_____ degree

03: ANS:=0.40

Problem 1

The speed of light is 3×10^8 m/s. The mass of an electron is 9.1×10^{-31} kg. Use Einstein formula to find the mass energy of an electron in eV. eV is so-called electron volt which is equal to 1.6×10^{-19} J. (01小題)

Problem 2

A planet is approximately a sphere of radius 4.2×10^6 m . Its mass is 1.5×10^{24} . Calculate (a)its surface area and (b)its average mass density. (

(a)its surface area=_____m²

05: ANS:=2.217E14

(b)its average mass density=_____ kg/m³

06: ANS:=4.833E3

```
#-----a planet R,C,S,V,D
```

```
M=1.5E24
```

```
R=4.2E6
```

```
C=2*pi*R
```

```
S=4*pi*R**2
```

```
V=4*pi*R**3/3
```

```
D=M/V
```

```
C= 2.639e+07
```

```
S= 2.217e+14
```

```
V= 3.103e+20
```

```
D= 4.833e+03
```

Problem 2

The distance between Sun and Earth is 1.5×10^{11} m, this is so-called AU, astronomical unit. The travelling distance of light in a year is called light year(ly). Answer the following questions. (02/小題)

(a)The distance between Sun and nearest star is 4.2 ly=_____ AU

07: ANS:=264902.4

```
dly,dau= 4.2 264902.4  
d= 1.200e+26 9.461e+15 1.268e+10
```

(b)If the light from a very distant galaxy 1.2×10^{26} m away, how long does it take for light to travel?
the time=_____ years

08: ANS:=1.268E10

```
# distance of AU and Light Year
```

```
au=1.5E11
```

```
ly=3E8*365*86400
```

```
dly=4.2
```

```
dau=dly*ly/au
```

```
print('dly,dau=',dly,dau)
```

```
dm=1.2E26
```

```
dly=dm/ly
```

```
print('d= {0:.3e} {1:.3e} {2:.3e}'.format(dm,ly,dly))
```


Problem 3

太陽的半徑大約是696340 km，質量大約是 2×10^{30} kg，其中有4分之3的質量是氫原子。請回答下列問題。(04小題)

(a) 地球的半徑是6400公里，請問太陽的體積約是地球體積的_____倍。

09: ANS:=1.288E6

(b) 一個氫原子的質量為 1.67×10^{-27} kg，請問太陽有_____個氫原子。

10: ANS:=0.898E57

(c) 我們可以利用太陽的質量和太陽每秒鐘消耗於核融合反應的質量，來推估太陽發光發熱的壽命有多長。已知太陽總質量的10%會進行核融合反應，太陽每秒鐘進行核融合反應所消耗的質量為 6.35×10^{11} kg，請利用這些數據來估計太陽可發光發熱的時間長度=_____秒。

11: ANS:=3.15E17

(d) 太陽可發光發熱的時間長度=_____年

12: ANS:=1E10

$$R = 6.35 \times 10^{11} \text{ kg/s}$$

$$N = \frac{2 \times 10^{30} \times 0.75}{1.67 \times 10^{-27}} = 0.898 \times 10^{57}$$

$$\frac{2 \times 10^{29}}{6.35 \times 10^{11}} = \underline{3.15 \times 10^{17} \text{ s}}$$

$$\frac{3.15 \times 10^{17}}{365 \times 86400} = 9.99 \times 10^9 \approx 100 \times 10^8 \text{ 年}$$

$$\frac{V_{sun}}{V_E} = \left(\frac{R_s}{R_E}\right)^3 = \left(\frac{0.696 \times 10^6}{6400}\right)^3 = 1.286 \times 10^9$$
$$M = 2 \times 10^{30}$$
$$m = 1.67 \times 10^{-27}$$

Problem 3

太陽的直徑大約是 1.392×10^6 km，質量大約是 2×10^{30} kg，其中有4分之3的質量是氫原子。請回答下列問題。(04小題)

(a) 地球的半徑是6400公里，請問太陽的體積約是地球體積的_____倍。

$$\frac{V_{sun}}{V_E} = \left(\frac{R_s}{R_E}\right)^3 = \left(\frac{0.696 \times 10^6}{6400}\right)^3 = 1.286 \times 10^9$$

09: ANS: = 1.286E9

(b) 一個氫原子的質量為 1.67×10^{-27} kg，請問太陽有_____個氫原子。

$$M = 2 \times 10^{30}$$
$$m = 1.67 \times 10^{-27}$$

10: ANS: = 1.2E57

(c) 我們可以利用太陽的質量和太陽每秒鐘消耗於核融合反應的質量，來推估太陽發光發熱的壽命有多長。已知太陽總質量的10%會進行核融合反應，太陽每秒鐘進行核融合反應所消耗的質量為100公噸，請利用這些數據來估計太陽可發光發熱的時間長度 = _____ 秒。

$$N = \frac{M}{m} = \frac{2 \times 10^{30}}{1.67 \times 10^{-27}} = 1.20 \times 10^{57}$$

11: ANS: = 3.15E17

(d) 太陽可發光發熱的時間長度 = _____ 年

12: ANS: = 1E10

$$\frac{(2 \times 10^{30} \times 10\%)}{6.35 \times 10^{11}} = 3.15 \times 10^{17} \text{ (s)}$$
$$= \frac{3.15 \times 10^{17}}{365 \times 86400} = 1 \times 10^{10} \text{ 年}$$

$$V_{sun} = \frac{4}{3} \pi (0.696 \times 10^9)^3$$
$$= 1.41 \times 10^{27} \text{ m}^3$$

$$v = \frac{4}{3} \pi (0.05 \times 10^9)^3 = 5.236 \times 10^{31} \text{ m}^3$$

$$V_{sun} = N v = 1.20 \times 10^{57} \times 5.236 \times 10^{31}$$
$$= 6.283 \times 10^{26} \text{ m}^3$$

$$\frac{V_{sm}}{V_E} = \left(\frac{R_s}{R_E}\right)^3 = \left(\frac{0.696 \times 10^6}{6400}\right)^3 = 1.286 \times 10^9 \quad \left(\frac{2 \times 10^{30} \times 10^{27}}{6.35 \times 10^{11}}\right) = 3.15 \times 10^{17} \text{ (s)}$$

$$= \frac{3.15 \times 10^9}{365 \times 86400} = 1 \times 10^{10} \frac{\text{s}}{\text{y}}$$

$$V_{sm} = \frac{4}{3} \pi (0.696 \times 10^9)^3$$

$$= 1.41 \times 10^{27} \text{ m}^3$$

$$M = 2 \times 10^{30}$$

$$m = 1.67 \times 10^{-27}$$

$$N = \frac{M}{m} = \frac{2 \times 10^{30}}{1.67 \times 10^{-27}} = 1.20 \times 10^{57}$$

$$v = \frac{4}{3} \pi (0.05 \times 10^{-9})^3 = 5.236 \times 10^{-31} \text{ m}^3$$

$$V_{sm} = N v = 1.20 \times 10^{57} \times 5.236 \times 10^{-31}$$

$$= 6.283 \times 10^{26} \text{ m}^3$$

Problem 4

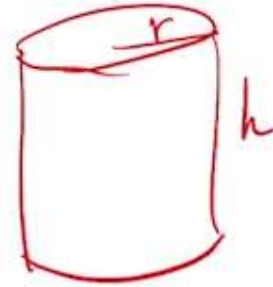
一個圓柱體底面的圓半徑為 r ，圓柱體的高度為 h ，請計算：(02小題)

(a) 這個圓柱體的體積 = _____ $[r, h]$ $\pi = \text{pi}$

17: ANS: = $\text{pi} * r^{**2} * h$

(b) 圓柱體的表面積 = _____

18: ANS: = $2 * \text{pi} * r^{**2} + 2 * \text{pi} * r * h$



$$V = (\pi r^2) h$$

$$S = (\pi r^2) \times 2 + (2\pi r) h$$

考慮一個圓錐體底面的半徑為 r ，圓錐的高度為 h 。

(a) 這個圓錐體的體積 = _____ $[r, h]$

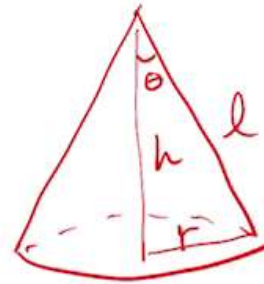
19: ANS: = $\frac{1}{3} * \text{pi} * r^{**2} * h$

(b) 圓錐體的表面積 = _____ $[r, h]$

20: ANS: = $\text{pi} * r^{**2} + \text{pi} * r * \text{sqrt}(r^{**2} + h^{**2})$

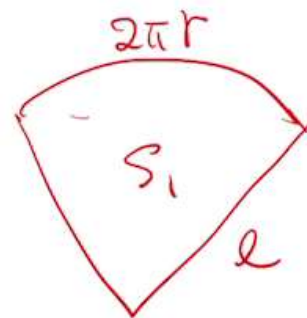
(c) 圓錐體的頂角為 θ ， $\cos(\theta) =$ _____ $[r, h]$

21: ANS: = $\frac{h}{\text{sqrt}(r^{**2} + h^{**2})}$



$$V = \frac{1}{3} A h$$
$$= \frac{1}{3} (\pi r^2) h$$

$$S = S_1 + \pi r^2$$
$$= \pi l r + \pi r^2$$
$$= \pi r (r + \sqrt{r^2 + h^2})$$

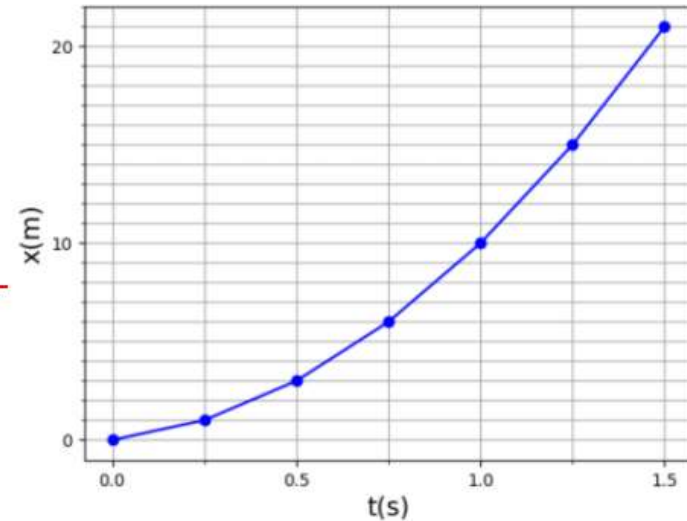


$$l = \sqrt{r^2 + h^2}$$
$$S_1 = \frac{2\pi r}{2\pi l} \cdot \pi l^2 = \pi l r$$

$$\cos \theta = \frac{h}{l} = \frac{h}{\sqrt{r^2 + h^2}}$$

Problem 5

An object is moving along the +x direction in a constant acceleration. The position of the object as a function of times is shown in the figure. Please calculate (a) the initial velocity v_0 at $t = 0$ and (b) the acceleration a . (c) Find the speed v of the object at the coordinate $x = 60$ m. (03小題)



(a) $v_0 =$ _____ m/s

16: ANS: = 2

(b) $a =$ _____ m/s²

17: ANS: = 16

(c) at $x = 60$ m, $v =$ _____ m/s

18: ANS: = 43.86

t	0	0.5	1	1.5
x	0	3	10	21

$$x(t) = 0 + v_0 t + \frac{1}{2} a t^2$$

$$3 = x(0.5) = 0 + 0.5v_0 + \frac{1}{2}a(0.5)^2$$

$$10 = x(1) = 0 + v_0 + \frac{1}{2}a(1)^2$$

$$\times 8: 24 = 4v_0 + a$$

$$\times 4: 40 = 4v_0 + 2a$$

$$\begin{aligned} x(1.5) &= 2(1.5) + \frac{1}{2}(16)(1.5)^2 \\ &= 3 + 18 = 21 \quad (v) \end{aligned}$$

$$v^2 = v_0^2 + 2as$$

$$= 2^2 + 2(16)(60)$$

$$= 1924$$

$$v = 43.82$$

Problem 6

A ball of mass 5 kg thrown up from the ground reaches a maximum height of 20 m. Find: (a) its kinetic energy K just before hitting the ground; (b) its displacement S between 0.5 and 2.5 s; (c) the time t at which it is 15 m above the ground and moving toward the ground. (03小題)

$$v^2 = v_0^2 + 2as$$

末速 $v=0$, $a=-9.8$, $s=20$

$$\begin{aligned} v_0^2 &= -2as = -2(-9.8)(20) \\ &= 392, \quad v_0 = 19.8 \text{ (m/s)} \end{aligned}$$

$$K = \frac{1}{2} m v_0^2 = \frac{1}{2} \times 5 \times 392 = 980 \text{ (J)}$$

(a) $K = \underline{\hspace{2cm}}$ J

19: ANS: = 980

(b) $S = \underline{\hspace{2cm}}$ m

20: ANS: = 10.2

(c) $t = \underline{\hspace{2cm}}$ s

21: ANS: = 3.03

$$v(0.5) = 19.8 - 9.8(0.5) = 14.9$$

$$S = 14.9 \times 2 + \frac{1}{2}(-9.8)(2)^2 = 10.2$$

$$[t = 2.5 - 0.5 = 2]$$

$$\begin{aligned} 15 &= 19.8t + \frac{1}{2}(-9.8)t^2 \\ 4.9t^2 - 19.8t + 15 &= 0 \end{aligned}$$

$$t = \frac{19.8 + \sqrt{19.8^2 - 4.9 \times 60}}{2(4.9)} = 3.03$$

Problem 6

Two balls are thrown toward each other: ball A at 16.0 m/s upward from the ground, ball B at 9.00 m/s downward from a roof 30.0 m high, one second later. (a) Where and (b) when do they meet? (c) Find the velocity of A (v_A) when they meet. Take the y-coordinate of the ground as $y = 0$ and the +y-axis is pointing upward. So the velocity of the ball is negative when it is moving downward. (03小題)

(a) the time they meet, $t = \underline{\hspace{2cm}}$ s

22: ANS: = 2.24

(b) the coordinate they meet, $y = \underline{\hspace{2cm}}$ m

23: ANS: = 11.3

(c) the velocity of A when they meet, $v_A = \underline{\hspace{2cm}}$ m/s

24: ANS: = -5.95

$$x_A(t) = 16t - 4.9t^2$$

$$x_B(t) = 30 - 9(t-1) - 4.9(t-1)^2$$

$$x_A = x_B \Rightarrow 16t = 30 - 9t + 9 + 9.8t - 4.9$$

$$15.2t = 34.1$$

$$t = 2.243 \text{ (s)}$$

$$\begin{aligned} x_A(2.243) &= 16(2.243) - 4.9(2.243)^2 \\ &= 11.24 \end{aligned}$$

$$v_A = 16 - 9.8(2.243) = -5.981$$

$v_A < 0$ \therefore A 在往下落的过程中与 B 相会

Problem 7

A particle is moving along a straight line and its position as a function of time is given by $x(t) = 5t^3 - 12t^2 + 6t - 8$. All quantities are in SI unit. Answer the following questions. (03小題)

(a) Find its velocity as a function of time, $v(t) = \underline{\hspace{2cm}}$ [t]

25: ANS: = $15t^2 - 12t + 6$

(b) Find its acceleration as a function of time, $a(t) = \underline{\hspace{2cm}}$ [t]

26: ANS: = $30t - 12$

(c) calculate the minimal speed of this particle, $v_{min} = \underline{\hspace{2cm}}$ m/s

27: ANS: = 3.6

$$x(t) = 5t^3 - 12t^2 + 6t - 8$$
$$v(t) = \frac{dx}{dt} = 15t^2 - 24t + 6 \quad \frac{dv}{dt} = 0 \Rightarrow a(t) = 0$$
$$a(t) = \frac{dv}{dt} = 30t - 24 \quad 30t - 24 = 0$$
$$t = \frac{24}{30} = 0.8$$

Problem 8

Answer the following questions. (04小題)

(a) The volume of 1 liter(L) is equal to 1000 cm^3 . A sphere has a volume of 8.5 L. What is its radius R ? $R = \underline{\hspace{2cm}} \text{ m}$

28: ANS:=0.1266

(b) the number of second in 4 centuries, 4 centuries = s

29: ANS:=1.261E10

(c) 1 amu = eV (電子伏特 · 1 eV = $1.6 \times 10^{-19} \text{ J}$)

30: ANS:=9.338E8

(d) The important equation of electromagnetic wave(電磁波): $v = \lambda f$, here v = speed of light, λ = wavelength, f = frequency. For a microwave of length 12 cm, $f = \underline{\hspace{2cm}} \text{ Hz}$

31: ANS:=2.5E9

Problem 9

一個恆星發光發熱的壽命有多長，可以透過測量恆星發光的功率和恆星的質量來計算，這個問題就透過一個簡單的例子來計算。我們假設有一個恆星其質量是 3.5×10^{30} kg，如果在距離這個恆星 2×10^{11} m上測量單位面積上恆星的光照強度為 4.8×10^3 W/m²，可以回推這個(a)恆星的發光功率。恆星利用核融合反應產生光和熱，在這個反應中能量的計算是利用愛因斯坦的質能互換公式 $E = mc^2$ 。恆星核融合反應的主要過程是由4個氫融合成一個氦，過程會有7%的質量損失，利用質能互換公式可以計算出釋放的能量。一個恆星並非所有的質量都能夠進行核融合反應，只有靠近核心的質量溫度夠高、壓力夠大，才能進行核融合反應。進行核融合反應的恆星質量約佔總體質量的10%。(b)請計算這個恆星在核融合的歲月中會損失的總質量以及(c)核融合反應所產生的總能量。利用恆星的發光功率和核融合反應所產生的總能量，來計算(d)這個恆星出發光發熱的壽命。(04小題)

(a)恆星的發光功率=_____ W

32: ANS:=2.413E27

(b)這個恆星在核融合的歲月中損失的總質量=_____ kg

33: ANS:=2.45E27

(c)核融合反應所產生的總能量=_____ J

34: ANS:=2.205E44

(d)這個恆星出發光發熱的壽命=_____ year

35: ANS:=2.90E9

Problem 10

An object is moving along the x-axis. The position of the object as a function of times is shown in the figure. It is found that the function of motion can be described as $x(t) = at^3 + bt^2 + ct + d$ with all quantities in SI units.

Please calculate a, b, c, d (05小題)

(a) $a =$ _____

36: ANS: = 1

(b) $b =$ _____

37: ANS: = -4

(c) $c =$ _____

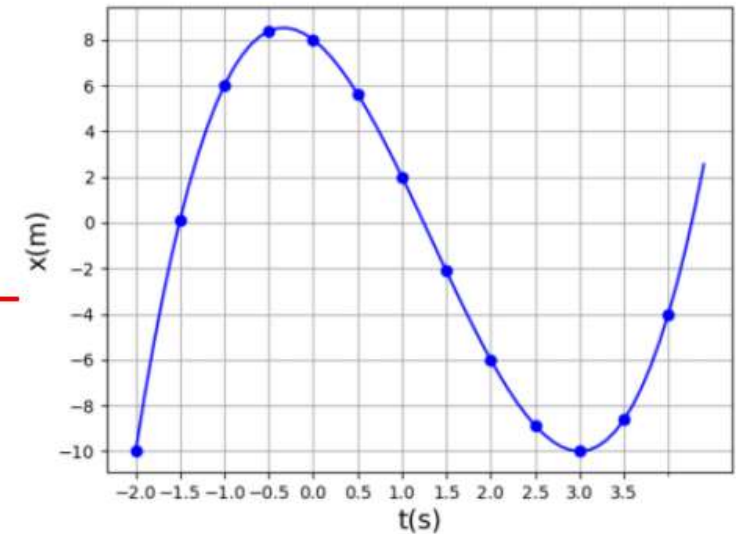
38: ANS: = -3

(d) $d =$ _____

39: ANS: = 8

(e) Find the time of zero velocity at the position on the **-x** side. time = _____ s

40: ANS: = 3.0



Problem 11

Consider 4 cases of one-dimensional motion with velocity vs. t shown in the figure. Answer the following questions.

Write the reason of your answer on the paper. (03小題)

(a) A ball is thrown up and then falls back to the ground. Which of the graphs in the figure best represents the variation of its velocity with time? Ans=_____ Enter 1,2,3,4 as the number of the figure you choose.

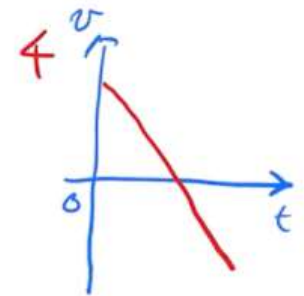
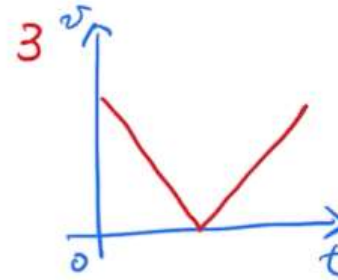
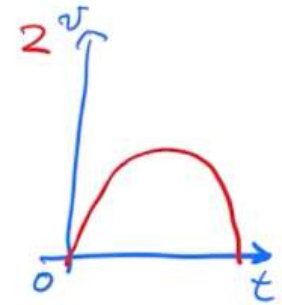
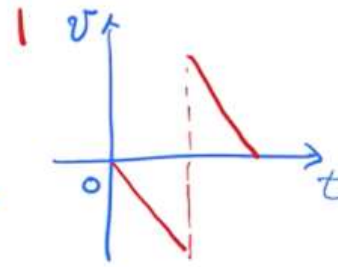
41: ANS:=4

(b) Which one of the figure represent a ball free fall from a height and bouncing on the ground? Ans=_____

42: ANS:=1

(c) Which one of the motion passes a time of zero acceleration? Ans=_____

43: ANS:=2




```
# Convert Liter to radius
V=3.2 # L (liter)
V=8.5 # L (liter)
R=(0.001*V*3/4/pi)**(1/3)
```

$\theta = s/R$ in radians, or $\sin(0.5'') = 0.5 \text{ AU}/1 \text{ parsec}$, give
1 parsec = $2.063 \times 10^5 \text{ AU}$.

Problem 3

A planet is approximately a sphere of radius 4.2×10^6 m . Its mass is 1.5×10^{24} . Calculate (a) its circumference, (b) its surface area and (c) its volume. (d)Its average mass density. (04 小題)

(a)its circumference=_____ m

06: ANS:=2.639E7

(b)its surface area=_____m²

07: ANS:=2.217E14

(c)its volume=_____m³

08: ANS:=3.103E20

(d)density=_____ kg/m

09: ANS:=4.833E3

```
import numpy as np
pi=np.pi

#-----a planet R,C,S,V,D
M=1.5E24
R=4.2E6
C=2*pi*R
S=4*pi*R**2
V=4*pi*R**3/3
D=M/V

print('C= {0:.3e}'.format(C))
print('S= {0:.3e}'.format(S))
print('V= {0:.3e}'.format(V))
print('D= {0:.3e}'.format(D))
```

```
C= 2.639e+07
S= 2.217e+14
V= 3.103e+20
D= 4.833e+03
```

Problem 3

The distance between Sun and Earth is 1.5×10^{11} m, this is so-called AU, astronomical unit. The travelling distance of light in a year is called light year(ly). Answer the following question. (02小題)

(a)the distance between Sun and nearest star is 4.2 ly=_____ AU

10: ANS:=264902.4

(b)If the light comes from a very distance galaxy 1.2×10^{26} m away, how long does it take to travel. The time=_____ years

11: ANS:=1.268E10

```
# distance of AU and Light Year
au=1.5E11
ly=3E8*365*86400
dly=4.2
dau=dly*ly/au
print('dly,dau=',dly,dau)
dm=1.2E26
dly=dm/ly
print('d= {0:.3e} {1:.3e} {2:.3e}'.format(dm,ly,dly))
```

```
dly,dau= 4.2 264902.4
d= 1.200e+26 9.461e+15 1.268e+10
```

Problem 4

若將本銀河系視為一個大圓盤，圓盤的直徑為10萬光年，圓盤的厚度2000光年，本銀河系的總質量為 0.8×10^{12} 太陽質量。太陽質量= 2×10^{30} kg. (04小題)

(a)本銀河系的總質量=_____ kg

13: ANS:=1.6E42

$$M = 0.8 \times 10^{12} \times 2 \times 10^{30} \\ = 1.6 \times 10^{42} \text{ kg}$$

(b)1光年=_____ m

14: ANS:=9.46E15

$$1 \text{ ly} = 3 \times 10^8 \times 365 \times 86400 = 9.46 \times 10^{15} \text{ m}$$

(c)本銀河系的體積=_____ m^3

15: ANS:=1.329E61

(d)本銀河系的密度=_____ kg/m^3

16: ANS:=1.204E-19

$$V = \pi r^2 d = \pi (5 \times 10^4)^2 (2 \times 10^3) \\ = 1.57 \times 10^{13} (\text{ly})^3 \\ = 1.57 \times 10^{13} (9.46 \times 10^{15})^3 \\ = 1.329 \times 10^{61} (\text{m}^3)$$

$$D = \frac{M}{V} = \frac{1.6 \times 10^{42} \text{ kg}}{1.329 \times 10^{61} \text{ m}^3} \\ = 1.204 \times 10^{-19} \text{ kg}/\text{m}^3$$

Problem 1

One molecule of water (H_2O) contains two atoms of hydrogen and one atom of oxygen. A hydrogen atom has a mass of 1.0 u and an atom of oxygen has a mass of 16 u. (a) What is the mass in kg of one molecule of water? (b) How many molecules of water are in the world's oceans, which have an estimated total mass of 1.4×10^{21} kg? (02小題)

(a) the mass of one molecule of water = _____ kg

01: ANS:=2.988E-26

(b) Number of molecules = _____

02: ANS:=4.685E+46

$u = 1.66\text{E-}27$ #kg

$Nu = 16 + 2$

$\mu = Nu * u$

$Mw = 1.4\text{E}21$ #kg

$Nmo = Mw / \mu$

`print('molecule mass=', mu)`

`print('Number of molecules=', Nmo)`

```
molecule mass= 2.988e-26
Number of molecules= 4.685408299866131e+46
```