GPN1-Q06

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東海大學 TUNGHAI UNIVERSITY

Problem 1

A wheel rotates with angular position given by $\theta = 5t^3 - 2t^2 - 4t + 3$. Find (a)the angle turned through by the wheel between times t = 3 and t = 5 seconds; (b)the average angular speed from t=3 to t=5; (c)the instantaneous angular speed at t=4; (d)the instantaneous angular acceleration at t=4. (04小題)

(a)From t=3 to t=5, $\Delta\theta$=______ rad

01: **ANS:=450**

(b)From t=3 to t=5, ω_{avg}=______ rad/s

02: **ANS:=225**

(c)$\omega(4)$=______ rad/s

03: **ANS:=220**

(d)$\alpha(4)$=______ rad/s^2

04: **ANS:=116**

$\theta(3) = 108$

$\theta(5) = 558$

$\Delta\theta = 558 - 108 = 450$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{450}{2} = 225$

$\theta(t) = 5t^3 - 2t^2 - 4t + 3$

$\omega = \frac{d\theta}{dt} = 15t^2 - 4t - 4$

$\alpha = \frac{d\omega}{dt} = 30t - 4$

Problem 1

What are the magnitudes of (a) the angular velocity, (b) the radial acceleration, and (c) the tangential acceleration of a spaceship taking a circular turn of radius 4000 km at a constant speed of 30000 km/h? (03小題)

(a)ω=____ rad./s

05: **ANS:=2.083E-3**

(b)radial acceleration, a_r=_____ m/s²

06: **ANS:=17.31**

(c)tangential acceleration, a_t=_____ m/s²

07: **ANS:=0**

$$v = 3\times10^4 \frac{km}{hr} = 8.33\times10^3 \; m/s$$

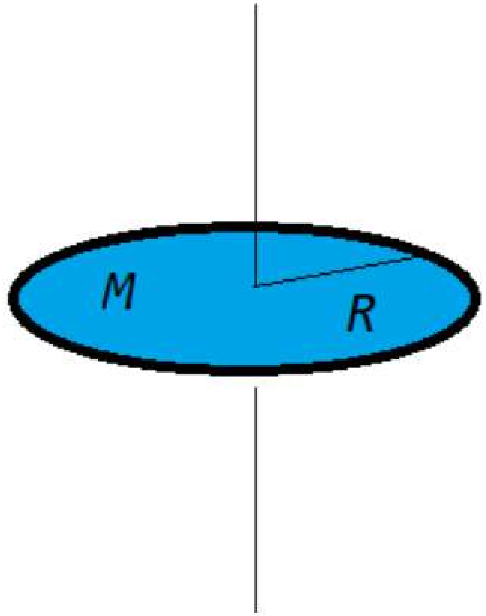
$$\omega = \frac{v}{r} = \frac{8.33\times10^3}{4000\times10^3} = 2.08\times10^{-3} \; rad/s$$

$$a_t = r\alpha = 0 \quad \left(\alpha = \frac{d\omega}{dt} = 0\right)$$

$$a_c = \frac{v^2}{r} = r\omega^2 = \frac{8.33^2}{4} = 17.36 \; m/s^2$$

Problem 2

A flywheel of radius 0.2 m and mass 10 kg with unknown initial angular speed requires 3 sec to rotate through 234 radian about the axis through its center (see figure). Its angular velocity at the end of this time is 108 rad/sec. It rotates in constant angular acceleration. Find the angular acceleration α; (b)its tangential speed at $t = 2$ s. (04小題)



(a)α=_____ rad/s^2

08: **ANS:=<u>20</u>**

(b)$v(2)$=_____ m/s

09: **ANS:=<u>17.6</u>**

(c)If we can treat the flywheel as a disk, its moment of inertial about the central axis in the figure=_______ kg.m^2

10: **ANS:=<u>0.2</u>**

(d)Followng (b,c), at $t = 2$ s, the rotational kinetic energy=______ J

11: **ANS:=<u>774.4</u>**

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$234 = \omega_0(3) + \frac{1}{2}\alpha(3)^2$

$\omega = \omega_0 + \alpha t$

$108 = \omega_0 + \alpha(3)$, $\omega_0 = 108 - 3\alpha$

$234 = 3(108 - 3\alpha) + \frac{9}{2}\alpha$

$\Rightarrow \alpha = 20$

$\omega(3) = \omega(2) + 20(3-2)$

$108 = \omega(2) + 20$, $\omega(2) = 88$

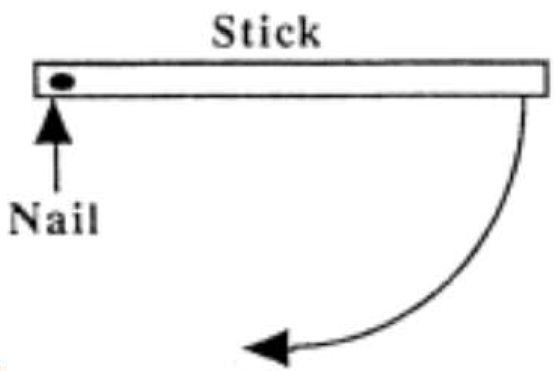
$v(2) = R\omega(2) = 0.2(88) = 17.6$ m/s

$I = \frac{1}{2}MR^2 = \frac{1}{2}(10)(0.2)^2 = 0.2$ (kg·m²)

$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.2)(88)^2 = 774.4$ (J)

Problem 3

A stick of length 1 m has a small hold drilled at one end so that it can swing freely on a nail in the wall. The stick is pulled to one side until it is horizontal, then released from rest. What is the angular speed of the stick as it swings through the vertical position? $\omega =$________rad/sec. In order to find the answer, you should find the change of the gravitational potential energy and the moment of inertia of the stick. (03小題)



(a)For the stick to swing from horizontal to vertical the change of potential energy ΔU=______ J

12: **ANS:=-0.98**

(b)The moment of inertia of the stick about the nail_______ kg.m^2

13: **ANS:=0.0667**

(c)the angular speed of the stick as it swings through the vertical position=______ rad/s

14: **ANS:=5.4**

$I = \frac{1}{3}ML^2 = 0.0667 \quad M=0.2,\ L=1$

$\Delta U = -Mg\left(\frac{L}{2}\right) = -(0.2)(9.8)\left(\frac{1}{2}\right) = -0.98$

$\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{3}ML^2\omega^2$

$= \frac{1}{2}(0.0667)\omega^2 = 0.98$

$\Rightarrow \omega = 5.42$

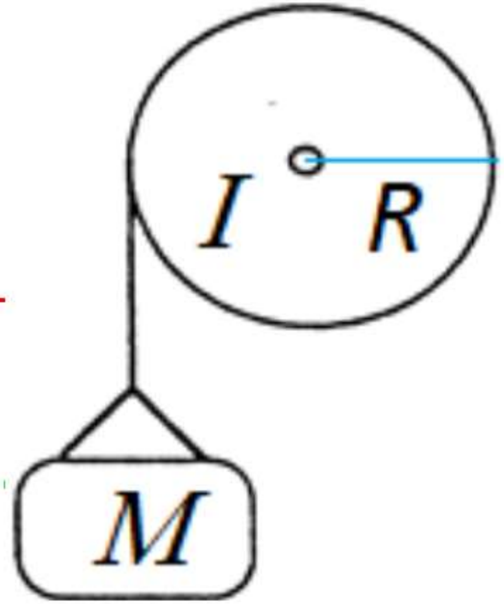
$\Delta K = -\Delta U$

$\frac{1}{6}\cancel{M}L^2\omega^2 = \frac{1}{2}\cancel{M}gL$

$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3(9.8)}{1}} = 5.42$

Problem 4

A heavy cylinder(圓柱體), with moment of inertial I and radius R, is mounted(安裝) on a horizontal axis with frictionless bearings(軸承). A light rope is tightly wrapped(纏繞) around the cylinder, so that no slipping(打滑) occurs. A block of mass M is hanging(懸掛) on rope, as shown. Derive an expression for his downward acceleration a. (01小題)



a=_______ [M,g,R,I]

15: **ANS:=(M*g*R**2)/(I+M*R**2)**

$$TR = I\alpha \rightarrow TR^2 = IR\alpha$$
$$= Ia$$

$$Mg - T = Ma$$

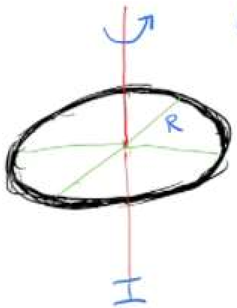
$$MgR^2 - TR^2 = MR^2 a$$

$$MgR^2 - Ia = MR^2 a$$

$$(I + MR^2)a = MgR^2$$

$$a = \frac{MgR^2}{I + MR^2}$$

A ring of mass M and radius 0.2 m is rotating about an axis passing through its center and perpendicular to the ring plane. The rotational kinetic energy is 24400 J and the angular speed is 602 rev/min(i.e., 602 rpm). The mass of the line(in green) connecting the ring and the axis is negligible. Find (a)the moment of inertia of the ring with respect to this axis and (b)the mass of the ring. (02小題)



A ring of mass M, radius R

"massless"

$K_{rot} = 24400\,J$

$\omega = 602\ rev/min$

$I = ?$

(a)moment of inertia, I=_____ kg.m^2

16: **ANS:=<u>12.28</u>**

(b)mass of the ring, M=_____ kg

17: **ANS:=<u>307</u>**

$$K_{rot} = \frac{1}{2} I \omega^2$$

$$24400 = \frac{1}{2} I \left(602 \cdot \frac{2\pi}{60}\right)^2$$

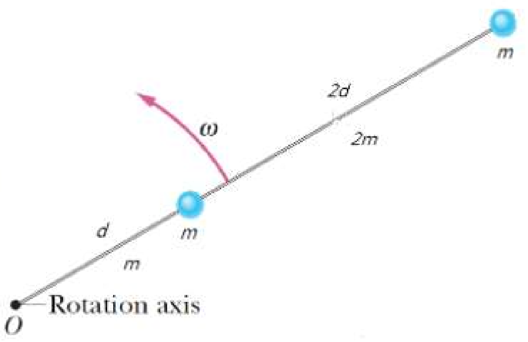
$$I = 12.28$$

$$I = MR^2, \quad M = \frac{I}{R^2} = \frac{12.28}{(0.2)^2}$$

$$= 307$$

Problem 5

The figure shows that there are two balls with a mass of m connecting two thin rods of different lengths to form a rigid body. One end of the entire rigid body is connected to the axis of rotation. The mass of the short rod is m and the length is d; The mass of the long rod is $2m$ and the length is $2d$. Calculate the moment of inertia of this rigid body relative to the axis of rotation. Support this rigid body is rotating at an angular rate ω=omega, please calculate the rotational kinetic energy. (02小題)



(a) rotational inertia=____ [m, d]

18: **ANS:=19*m*d**2**

(b) kinetic energy=_____ [m, d, omega]

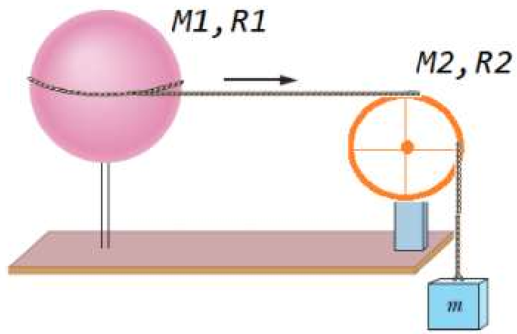
19: **ANS:=(19*m*d**2*omega**2)/2**

$$I = md^2 + m(3d)^2 + (\tfrac{1}{3})(3m)(3d)^2$$
$$= 10md^2 + 9md^2$$
$$= 19md^2$$

$$K_{rot} = \frac{1}{2} I \omega^2$$
$$= \frac{19}{2} md^2\omega^2$$

Problem 5

A uniform sphere of mass M1 and radius R1 can rotate about a vertical axis on frictionless bearings (see figure). A massless cord passes around the equator of the sphere, over a ring of mass M2, radius R2, and is attached to a small object of mass m. There is no friction on the ring's axle; the cord does not slip on the ring. The mass cross (2 perpendicular diameters of the ring) in the ring is negligible. What is the speed of the object when it has fallen H after being released from rest? (Hints: 1. Use energy considerations. 2. The moment of inertia of a sphere=$\frac{2}{5}MR^2$.) (01小題)



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the speed of the object, v=______ [M1,M2,R1,R2,m,H,g]

20: **ANS:=sqrt((m*g*h)/(1/2*m+1/5*M1+1/4*M2))**

$$\Delta U = -mgH$$

$$\Delta K = -\Delta U = mgH$$

$$\Delta K = \frac{1}{2}\left(\frac{2}{5}M_1R_1^2\right)\left(\frac{V}{R_1}\right)^2 + \frac{1}{2}\left(M_2R_2^2\right)\left(\frac{V}{R_2}\right)^2 + \frac{1}{2}mV^2$$

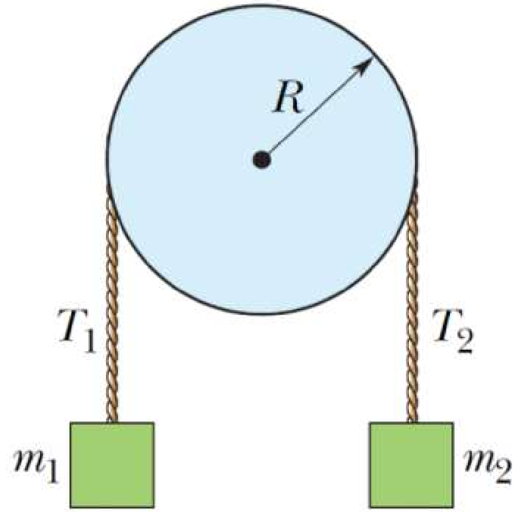
$$= v^2\left\{\frac{1}{5}M_1 + \frac{1}{4}M_2 + \frac{1}{2}m\right\}$$

$$= mgH$$

$$V = \sqrt{\frac{mgH}{\left(\frac{1}{5}M_1 + \frac{1}{2}M_2 + \frac{1}{2}m\right)}}$$

Problem 6

In the figure, block 1 has mass $m_1 = 0.35$ kg, block 2 has mass $m_2 = 0.5$ kg, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 0.2$ m. When released from rest, block 2 falls a distance $y = 1.25$ m in time $t = 4$ s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2 and (c) tension T_1? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia? (05小題)



(a)the acceleration a=______ m/s^2

21: **ANS:=0.1562**

(b)T_2=______ N

22: **ANS:=4.8219**

(c)T_1=______ N

23: **ANS:=3.4847**

(d)angular acceleration α=_______ rad/s^2

24: **ANS:=0.7812**

(e)moment of inertia of the pulley, I=______ kg.m^2

25: **ANS:=0.3423**

$y = \frac{1}{2}at^2$, $1.25 = \frac{1}{2}a(4)^2$

$a = 0.1562$

$T_2 = m_2(g-a) = 0.5(9.8-0.1562)$
$= 4.8219$

$T_1 = m_1(g+a) = 0.35(9.8+0.1562)$
$= 3.4847$

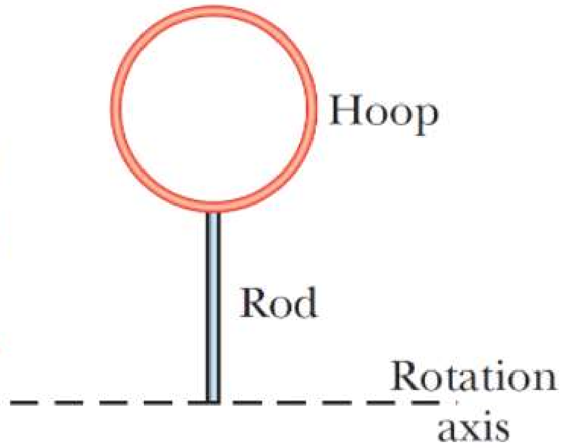
$(T_2 - T_1)R = I\alpha$

$\alpha = \frac{a}{R} = \frac{0.1562}{0.2} = 0.781$

$I = \frac{(4.8219-3.4847)(0.2)}{0.781} = 0.342$

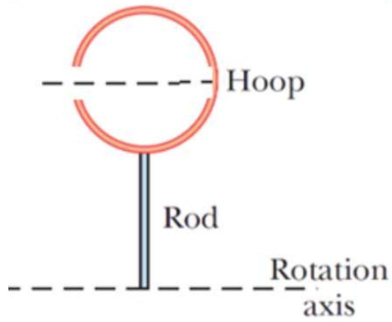
Problem 7

圖中顯示了一個細環（質量為 m ，半徑 $R = 0.150$ ）m和細桿（質量 m 和長度 $L = 2R$ ）的剛體。 剛體是直立的，但如果我們輕輕地輕推它，它將圍繞桿和箍平面內的水平軸旋轉穿過桿的下端。假設在這種輕推中給予剛體的能量可以忽略不計，當剛體通過顛倒（倒置）方向時，剛體繞旋轉軸的角速度是多少？ (03小題)



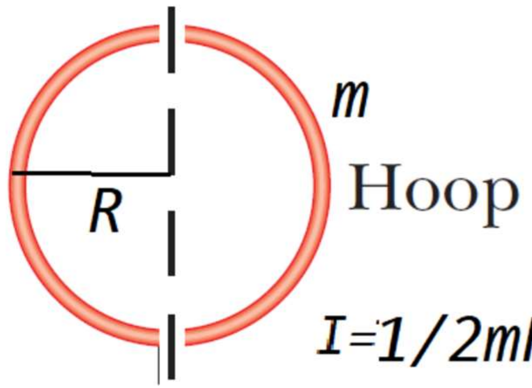
(a)剛體相對於轉動軸的轉動慣量=______ mR^2

26: **ANS:=10.83**



(b)剛體的質心座標與轉動軸之間的距離=_____ R

27: **ANS:=2**



$I=1/2mR^2$

(c)整個剛體轉動180度當圓環到達最低點處時剛體的轉動角速率=_______ rad/s

28: **ANS:=9.82**

the parallel axis theorem $L = 2R$

$$I = \frac{1}{12}mL^2 + m(L/2)^2 + \frac{1}{2}mR^2 + m(R+L)^2 = 10.83mR^2 ,$$

the center of mass is at

$$y = \frac{mL/2 + m(L+R)}{m+m} = 2R .$$

$$K = (2m)g(4R) \quad \Rightarrow \quad \omega = 9.82 \text{ rad/s} .$$

$$\Delta U = mgh = mg(4R)$$
$$\Delta K = -\Delta U$$
$$K = 1/2I\omega^2 = 1/2(10.83)mR^2\omega^2$$
$$= mg(4R)$$