GPN1-Q07

Problem 1

An automobile traveling at 48 m/s has tires of radius 0.5 m. (a) What is the angular speed of the tires about their axles? (b) If the car is brought to a stop uniformly(等加速度運動) in 30.0 complete turns of the tires (without skidding), what is the magnitude of the angular acceleration of the wheels? (c) How far does the car move during the braking? (05小題)

(a)the initial angular speed of the tires, ω_0=_____ rad/s

01: **ANS:=96**

(b)the magnitude of the angular acceleration, α=_____ rad/s^2

02: **ANS:=24.45**

(c)the distance the car moves=_____ m

03: **ANS:=94.25**

(d)the magnitude of the linear acceleration, a=_____ m/s^2

04: **ANS:=12.22**

(e)the time for the car come to rest=_____ s

05: **ANS:=3.927**

$\omega_0 = \frac{v_0}{R} = \frac{48}{0.5} = 96$ rad/s

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$0 = 96^2 + 2\alpha(30 \cdot 2\pi)$

$\alpha = -24.45$

$a = R\alpha = 0.5(-24.45) = -12.22$

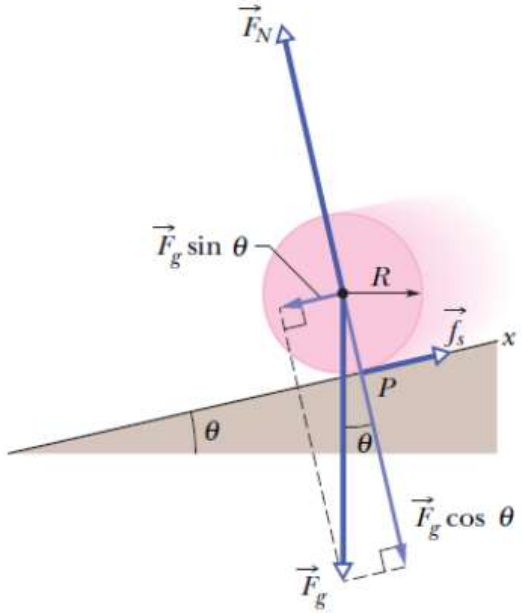
$S = R\theta = 0.5(30 \cdot 2\pi) = 94.25$

$\omega = \omega_0 + \alpha t$

$t = \frac{0-96}{-24.45} = 3.927$

Problem 2

A uniform ball(solid sphere,實心球體), of mass $M = 8.00$ kg and radius $R = 0.3$ m, rolls smoothly from rest down a ramp at angle 30.0° (see figure). The ball descends a vertical height $h = 2.0$ m to reach the bottom of the ramp. (a)What is its speed at the bottom? (b)What is the acceleration of the ball? (c) What is the magnitude of the frictional force on the ball as it rolls down the ramp? (d)What is the rotational kinetic energy of the ball at the bottom of the ramp? (07小題)



(a)its speed, v=_____ m/s

06: **ANS:=5.2915**

(b) acceleration, a=_____ m/s^2

07: **ANS:=3.5**

(c)the magnitude of the frictional force, f_s=_____ N

08: **ANS:=11.2**

(d) rotational kinetic energy=_____ J

09: **ANS:=44.8**

(e)the ratio of the rotational kinetic energy to the translational kinetic energy, K_{rot}/K_{tr}=_____

10: **ANS:=0.4**

(f)Following previous problem, if we replace the ball with a disk of mass 0.6 kg and radius 0.5 m, the acceleration=_____ m/s^2

11: **ANS:=3.267**

(g)if we replace the ball with a hoop of mass 0.6 kg and radius 0.5 m, the acceleration=_____ m/s^2

12: **ANS:=2.45**

$$a = R\alpha$$

$$Rf_s = I\alpha$$

$$mg\sin\theta - f_s = ma$$

$$mg\sin\theta - \frac{I}{R}\alpha = mR\alpha$$

$$\alpha = \frac{mg\sin\theta}{mR + \frac{I}{R}}$$

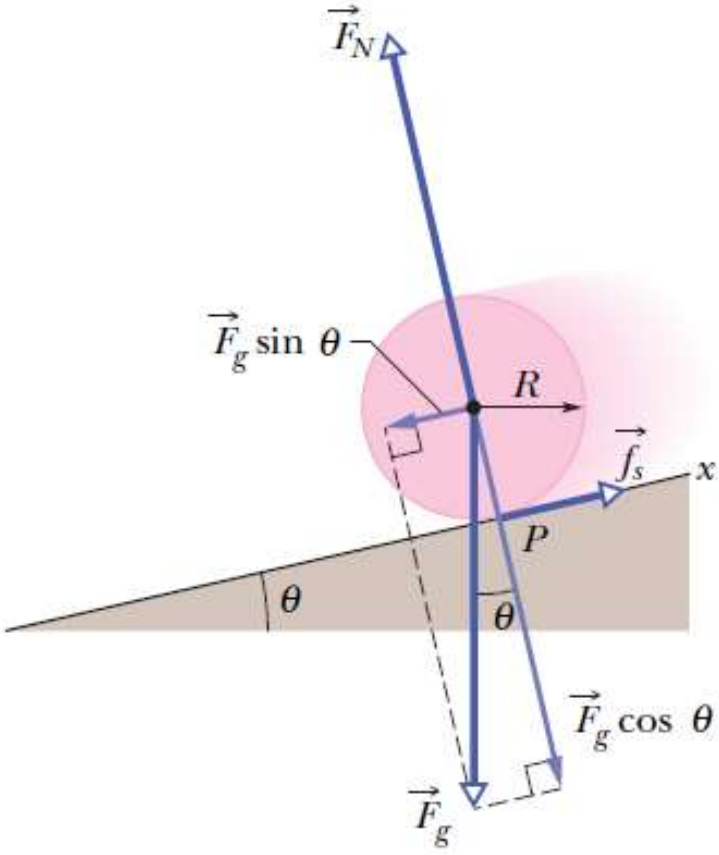
$$a = R\alpha = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

$$l = \frac{h}{\sin\theta}, \quad v^2 = v_0^2 + 2al$$

$$= 0 + 2(\quad)\frac{h}{\sin\theta} = \frac{2g}{1 + \frac{I}{mR^2}}$$

$$\frac{K_{rot}}{K_{tr}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \frac{I\omega^2}{MR^2\omega^2} = \frac{I}{MR^2}$$

$$I = \frac{2}{5}MR^2 \longrightarrow \text{ratio} = \frac{2}{5}$$



$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

$$\beta = \frac{I}{MR^2} = \begin{cases} \frac{2}{5}, & \text{solid sphere} \\ \frac{1}{2}, & \text{disk} \\ 1, & \text{Hoop} \end{cases}$$

Problem 3

A turntable with a moment of inertia of 0.012 kg.m^2 rotates freely at 2 rad/s. A circular disk of mass 200 g and diameter 30 cm, and initially not rotating, slips down a spindle and lands on the turntable. (a) Find the new angular speed. (b) What is the change in kinetic energy? (02小題)

(a)new angular speed=_____ rad/s

13: **ANS:=1.68**

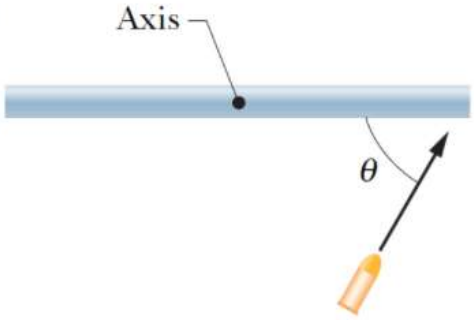
(b)change in kinetic energy=_____ .

14: **ANS:=-4E-3**

(a) $L_1 = I_1\omega_1 = 0.024$, $L_2 = I_2\omega_2 = 0.0143\omega_2$.
From $L_2 = L_1$, we find $\omega_2 = 1.68$ rad/s.
(b) $K_i = 0.024$ J, $K_f = 0.020$ J, thus $\Delta K = -4\text{x}10^{-3}$ J

Problem 3

一根長度為 0.500 m、質量為 4.00 kg 的均勻細棒可以在水平面上繞通過其中心的垂直軸旋轉。 一個快速運動的子彈(m=3.0 g)在細棒的旋轉面上以一個斜向的射角射入細棒的一端，細棒原先處於靜止狀態。 從上面看，子彈的路徑與細棒成角度 $\theta = 60.0°$（見圖）。 如果子彈停留在桿中並且在碰撞後，細棒的角速度為 10 弧度/秒，那麼子彈在撞擊前的速度是多少？ (02小題)



(a)碰撞之前子彈相對於旋轉軸的角動量=_____ kg.m^2/s

15: **ANS:=0.835**

(b)子彈的初速度=_____ m/s

16: **ANS:=1286**

$L = L' = I'\omega'$

$I' = \frac{1}{12}(4)(0.5)^2 + (0.003)(0.25)^2$

$= 0.0835$

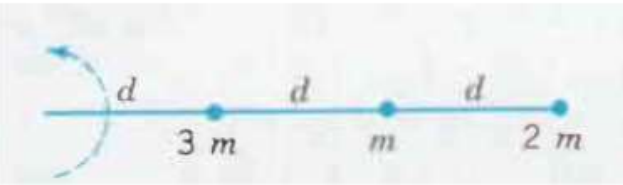
$L = (0.0835)(10) = 0.835$ kgm^2/s

$= mv(\frac{L}{2})\sin 60° = (0.003)(0.25)(\frac{\sqrt{3}}{2})v$

$v = \frac{0.835}{(0.003)(0.25)(\frac{\sqrt{3}}{2})} = 1286$ m/s

Problem 4

Three point particles with masses $3m, m, 2m$ are connected by rods of length d and mass u as shown in the figure. The system rotates at angular velocity w about one end. What are (a)the moment of inertia and (b)the angular momentum of the system? (02小題)



(a)I=_____ [m,u,d,w]

17: **ANS:=(9*u+25*m)*d**2**

(b)angular momentum, L=_____ [m,u,d,w]

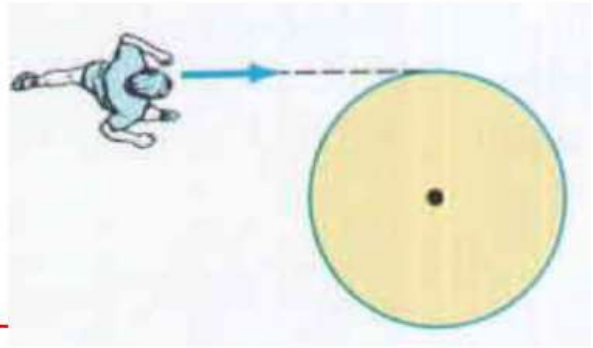
18: **ANS:=(9*u+25*m)*d**2*w**

(b) $L = [Md^2/3 + M(d^2/12 + (1.5d)^2) + M(d^2/12 + (2.5d)^2)]\omega +$

$+ 25md^2\omega = (9M + 25m)d^2\omega$

Problem 5

A 60-kg person runs with a velocity of 5 m/s along a tangent to a stationary circular platform, of radius 3 m and mass 100 kg, and jumps on (see figure). The platform (a disk) can rotate about a vertical axis. Find: (a) the angular velocity after the person jumps on; (b) the loss in mechanical energy. (02小題)



(a) angular velocity, ω=_____ rad/s

19: **ANS:=0.909**

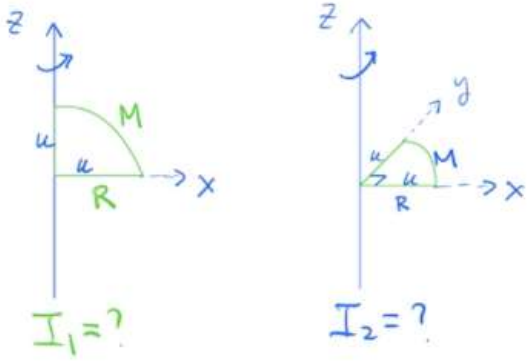
(b) the loss in mechanical energy, $|\Delta E|$=_____ J

20: **ANS:=341**

(a) $L = muR = 900\ kg.m^2/s$; $L_f = (1/2\ MR^2 + mR^2)\omega = 990\omega$,
thus $\omega = 0.909$ rad/s.
(b) $1/2\ mv^2 = 750$ J, $K_f = 1/2(1/2\ MR^2 + mR^2)\omega^2 = 409$ J
Thus $\Delta K = -341$ J

Problem 5

有一個剛體由一個1/4圓弧和兩個竿子共同組成，竿子的質量為u長度是R，1/4圓弧的半徑是R質量為M，請計算這個剛體相對於圖中不同的轉動軸的轉動慣量。 (02小題)

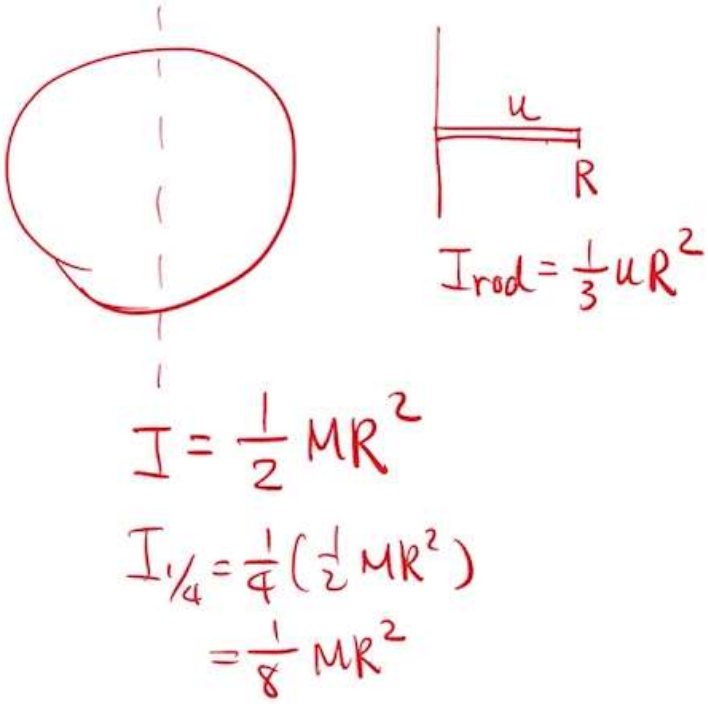


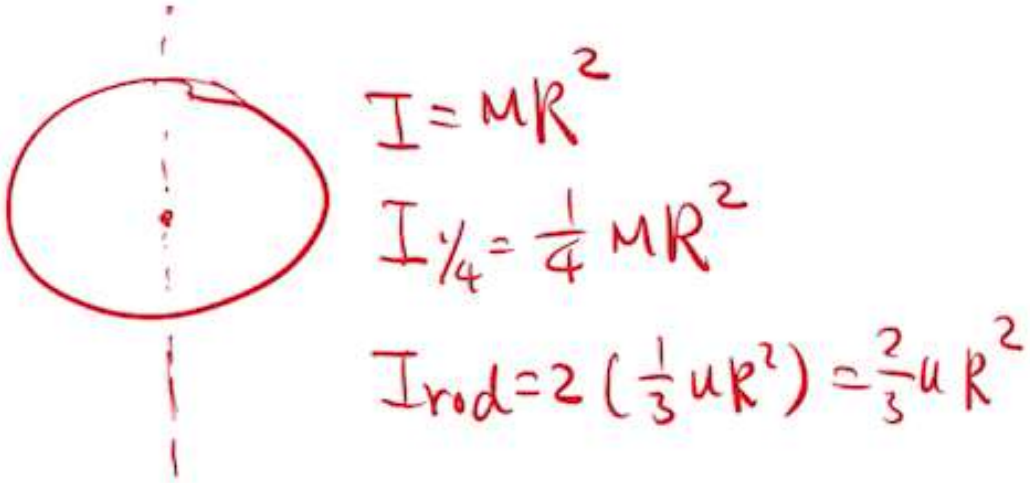
(a)I_1=_____ [u,M,R]

21: **ANS:=(1/8*M+1/3*u)*R**2**

(b)I_2=_____ [u,M,R]

22: **ANS:=(1/4*M+2/3*u)*R**2**





Problem 6

Two identical hoops, with rotational of mass M and radius R, roll without sliding across a horizontal floor with the same speed and then up inclines. Hoop A rolls up its incline without sliding. On the other hand, hoop B rolls up a frictionless incline. Otherwise the inclines are identical. Hoop A reaches a height 12 cm above the floor before rolling down again. (02小題)

(a)Hoop B reaches a height above the floor of h, h=_____ cm

23: **ANS:=6**

(b)Calculate the angular speed of hoop A when it reaches the height 6 cm. ω=_____ rad/s

24: **ANS:=0.707**

For A: $I_{hoop} = MR^2$

$K = \frac{1}{2}Mv_0^2 + \frac{1}{2}(MR^2)\left(\frac{v_0}{R}\right)^2$

$= Mv_0^2 = U = Mg(12)$

$v_0^2 = 12g$

For B: at h', $K = K_{rot} = \frac{1}{2}Mv_0^2$

$U + K_{rot} = Mgh' + \frac{1}{2}Mv_0^2 = Mv_0^2$

$h' = \frac{v_0^2}{2g} = \frac{12g}{2g} = 6$

For A, at $h = 6$ cm

$U = Mg(6) + K = 12Mg$

$K = 6Mg$

$\left(\frac{1}{2}Mv^2\right)\times 2 = 6Mg = \frac{1}{2}Mv_0^2$

$2v^2 = v_0^2$, $\frac{v}{v_0} = \frac{1}{\sqrt{2}}$

Problem 6

一個0.5公斤的棒球，以60度角，20 m/s的初速度射出，請計算$t = 2$ s的時候棒球相對於發射點(設為座標原點)的角動量；此時重力相對於原點對棒球的力矩。 (02小題)

(a)|角動量|=_____ kg.m^2/s

25: **ANS:=98**

(b)|力矩|=_____ N.m

26: **ANS:=98**

$v_{0x} = 20\cos 60^\circ = 10$

$v_{0y} = 20\sin 60^\circ = 10\sqrt{3} = 17.32$

$t=2,\ x = v_{0x}(2) = 20$

$y = 17.32(2) - \frac{1}{2}(9.8)(2)^2$

$= 15.04$

$\vec{p} = m\vec{v} = (0.5)(10, -2.28, 0) = (5, -1.14, 0)$

$\vec{L} = \vec{r} \times \vec{p} = (20, 15.04, 0) \times (5, -1.14, 0)$

$= -98\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} = (20, 15.04, 0) \times (0, -4.9, 0)$

$= (0, -98, 0)$

$v_y = v_{0y} - gt = 17.32 - (9.8)(2) = -2.28$