GPN1-Q08

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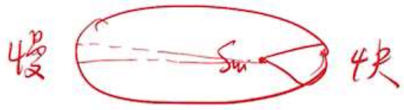
Problem 1

Answer the following questions:(**PART.1**) (09小題)

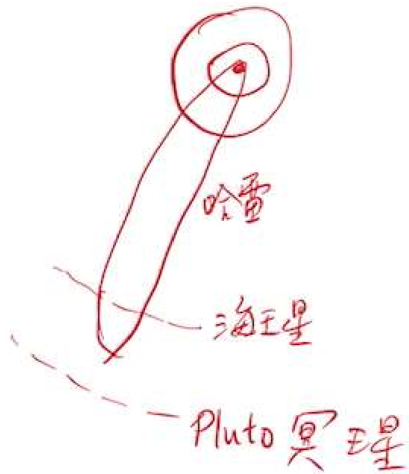
(a)一個體重60公斤的的男子在地面上受到的重力約為_____牛頓？ $F=W=mg=60\times9.8=588$

01: **ANS:=588**



(b)行星運動時，在近日點的速度？ 1=faster, 2=slower

02: **ANS:=1**

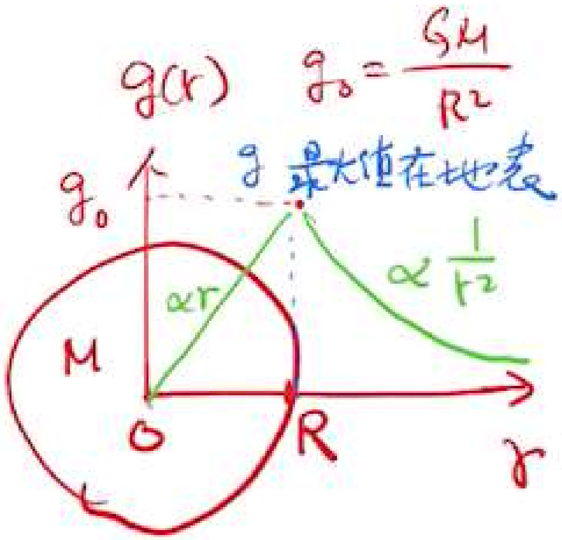


(c)哈雷彗星繞日為何種軌道？ 1=拋物線; 2=雙曲線; 3=圓曲線; 4=橢圓曲線; 5=雙螺旋曲線

03: **ANS:=4**

(d)請問下列哪一個地點的重力加速度最大？假設地球的半徑為R。1=距離地心0.25R處; 2=距離地心0.5R處; 3=地表 (距離地心R處); 4=距離地心1.5R處; 5=距離地心2R處; 6=距離地心0.1R處

04: **ANS:=3**



(e)如果地球的半徑縮小為原來的一半，但地球的質量不改變，則地球的重力加速度將為_______ m/s^2

05: **ANS:=39.2**

$$g=\frac{GM}{R^2} \quad R\to\frac{R}{2} \Rightarrow g'=\frac{GM}{(\frac{R}{2})^2}=4g$$

$$=4(9.8)=39.2$$

(f)克卜勒行星運動第二定律又稱為等面積定律，這個定律對應到軌道運動中必須滿足的哪一個定律？1=動量守恆定律; 2=動能守恆定律; 3=力學能守恆定律; 4=角動量守恆定律; 5=牛頓運動第二定律; 6=虎克定律; 7=庫倫定律; 8=安培定律; 9=作用力與反作用力定律

06: **ANS:=4**

(g)火星的質量為0.64×10^{24} kg，半徑為3.4×10^{6} m，請利用牛頓的重力定律計算火星的重力加速度約 _____ m/s^2。 重力常數$G = 6.67 \times 10^{-11}$

$$g_{Mars} = \frac{GM_{Mars}}{R^2_{Mars}} = \frac{6.67\times10^{-11}\cdot0.64\times10^{24}}{(3.4\times10^{6})^2} \simeq 3.7$$

07: **ANS:=3.7**

(h)有一個衛星在距離地表160公里高的地方繞著地心進行圓週運動，請計算這個衛星運動的速度v=______ m/s

08: **ANS:=7820**

(i)follow (h), 請計算這個衛星繞地心圓週運動的週期T=______ s

09: **ANS:=5250**

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \mathrm{m^3/s^2\cdot kg})(5.98\times10^{24}\ \mathrm{kg})}{6.53\times10^{6}\ \mathrm{m}}} = 7.82\times10^{3}\ \mathrm{m/s}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.53\times10^{6}\ \mathrm{m})}{7.82\times10^{3}\ \mathrm{m/s}} = 5.25\times10^{3}\ \mathrm{s}.$$

$$\frac{5250}{3600} = 1.46\ (\mathrm{hr})$$

Problem 2

A ball of mass 7.2 kg is orbiting Earth at an altitude h of 350 km in a circular orbit. Mass and radius of Earth are 6E24 kg and 6.4E6 m, respectively. G=6.67E-11. (07小題)

$r = R + h = 6370 \text{ km} + 350 \text{ km} = 6.72 \times 10^6 \text{ m},$

(a)the gravitational potential energy of the ball in orbit, U=_____ J

$$U = -\frac{GMm}{r} \qquad K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r},$$

10: **ANS:=-4.28E8**

(b)the kinetic energy of the ball in orbit, K=_____ J

$$E = -\frac{GMm}{2r} \qquad E = K + U = -\frac{GMm}{2r}$$

11: **ANS:=2.14E8**

(c)the mechanical energy of the ball in orbit, E=_____ J

$$= -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(7.20 \text{ kg})}{(2)(6.72 \times 10^6 \text{ m})}$$

$$= -2.14 \times 10^8 \text{ J} = -214 \text{ MJ}. \qquad \text{(Answer)}$$

12: **ANS:=-2.14E8**

(d)the speed of the ball in orbit, v=_____ m/s

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{2K}{m}}$$

$$v = \left(\frac{6.67\times10^{-11} \times 6\times10^{24}}{6.72\times10^6}\right)^{1/2} = 7.72\times10^3$$

13: **ANS:=7.72E3**

(e)the mechanical energy of the ball on Earth surface, E=_____ J

On Earth, $r = R_E, U = -\dfrac{GMm}{R_E}$

14: **ANS:=-4.375E8**

(f)the work to be done on the ball to place it into orbit=______ J

work, $W = \Delta E = E(R_E + h) - E(R_E)$

$\Delta E = -2.14\times10^8 - (-4.375\times10^8)$

$= 2.235\times10^8 \text{ (J)} = W$

15: **ANS:=2.235E8**

(g)the escape speed of the ball on the surface of Earth=_____ m/s

16: **ANS:=1.12E4**

$$v_{esc} = \sqrt{\frac{2GM}{R_E}} = \sqrt{\frac{(6.67\times10^{-11})(6\times10^{24})\times2}{6.4\times10^6}} = 1.118\times10^4 \text{ m/s}$$

Problem 3

(a)An asteroid, headed directly toward Earth, has a speed of 12 km/s relative to the planet when the asteroid is 10 Earth radii from Earth's center. Neglecting the effects of Earth's atmosphere on the asteroid, find the asteroid's speed v_f when it reaches Earth's surface. (02小題)

v_f=_____ m/s

17: **ANS:=1.6E4**

$$K_f + U_f = K_i + U_i.$$

$$\frac{1}{2}mv_f^2 - \frac{GMm}{R_E} = \frac{1}{2}mv_i^2 - \frac{GMm}{10R_E}.$$

(b)A very massive object exists at the center of our Milky Way galaxy. The evidence is the observation of the S2 star orbiting the unseen object with a period of 15.2 years and with semi-major axis of $a = 1.42 \times 10^{14}$ m. Find the mass of the object. mass=______ kg

18: **ANS:=7.35E36**

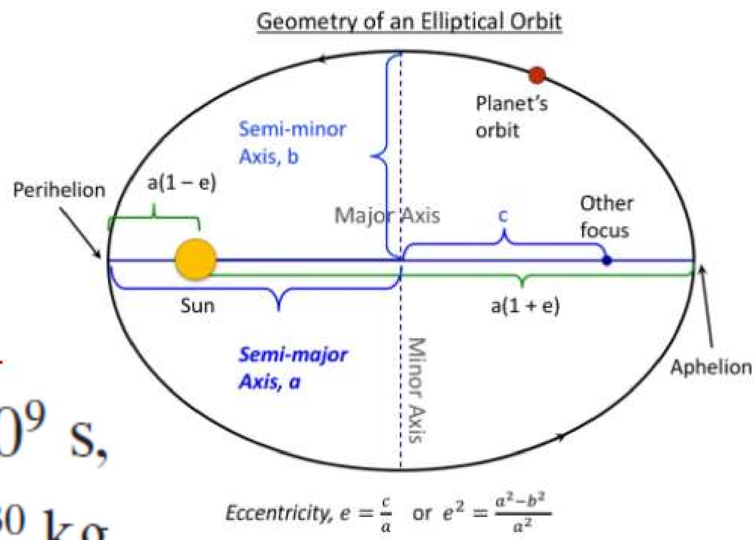
$$v_f^2 = v_i^2 + \frac{2GM}{R_E}\left(1 - \frac{1}{10}\right)$$
$$= (12 \times 10^3 \text{ m/s})^2 + \frac{2(6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24} \text{ kg})}{6.37 \times 10^6 \text{ m}} 0.9$$
$$= 2.567 \times 10^8 \text{ m}^2/\text{s}^2,$$

$$v_f = 1.60 \times 10^4 \text{ m/s}$$

$$M = \frac{4\pi^2 a^3}{GT^2}$$
$$= \frac{4\pi^2(1.42 \times 10^{14} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)[(15.2 \text{ y})(3.16 \times 10^7 \text{ s/y})]^2}$$
$$= 7.35 \times 10^{36} \text{ kg}.$$

Problem 4

Comet Halley orbits the Sun with a period of 76 years and, in 1986, had a distance of closest approach to the Sun(see figure), its perihelion distance R_p, of 8.9×10^{10} m which is between the orbits of Mercury and Venus. What is the comet's farthest distance from the Sun, which is called its aphelion distance R_a? (03小題)



(a)the semi-major axis of the orbit, a=_______ m

2.4×10^9 s,

19: **ANS:=2.7E12**

$$a = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}.$$

1.99×10^{30} kg.

$a = 2.7 \times 10^{12}$ m.

(b)R_a=________ m

20: **ANS:=5.3E12**

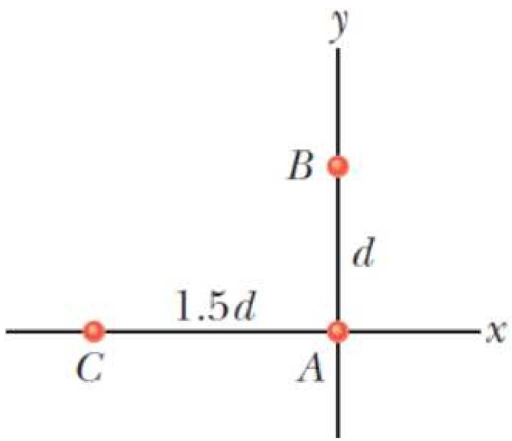
(c)It is observed that the speed of the comet is $v_p = 5.5 \times 10^4$ m/s at perihelion, $r = R_p$, the nearest point. Find its speed at aphelion. v_a=_______ m/s

21: **ANS:=923**

$$r_p v_p = r_a v_a \Rightarrow v_a = \frac{(8.9E10)(5.5E4)}{5.3E12} = 923$$

Problem 5

In the figure, three point particles are fixed in place in an xy plane. Particle A has mass m, particle B has mass 2m, and particle C has mass 3m. A fourth particle D, with mass 4m, is to be placed near the other three particles. In terms of distance d, at what (a) x coordinate and (b) y coordinate should particle D be placed so that the net gravitational force on particle A from particles B, C, and D is zero? (02小題)



(a)x=_____ d

22: **ANS:=0.716**

(b)y=______ d

23: **ANS:=-1.07**

題目要我們計算D質點放置的座標
可使A質點受B, C, D的重力合力為0

$\vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} = 0$

$\vec{F}_{AB} = \frac{G(2m^2)}{d^2}\hat{j} = 2f_0\hat{j}, \quad f_0 = \frac{Gm^2}{d^2}$

$\vec{F}_{AC} = \frac{G(3m^2)}{(1.5d)^2}(-\hat{i}) = -1.333 f_0\hat{i}$

$\vec{F}_{AD} = -(\vec{F}_{AB} + \vec{F}_{AC}) = 1.333 f_0\hat{i} - 2f_0\hat{j}$

設 $\vec{r}_D = (x, y) = (x_0, y_0)d$

$r_D^2 = (x_0^2 + y_0^2)d^2$

$|\vec{F}_{AD}| = \sqrt{1.333^2 + 2^2}\, f_0 = 2.404 f_0$

$= \frac{G(4m^2)}{(x_0^2+y_0^2)d^2} = \frac{4}{x_0^2+y_0^2} f_0$

$\frac{4}{x_0^2+y_0^2} = 2.404, \quad x_0^2 + y_0^2 = 1.664$

$\frac{y_0}{x_0} = \frac{F_{AD,y}}{F_{AD,x}} = \frac{-2f_0}{1.333 f_0} = -1.5$

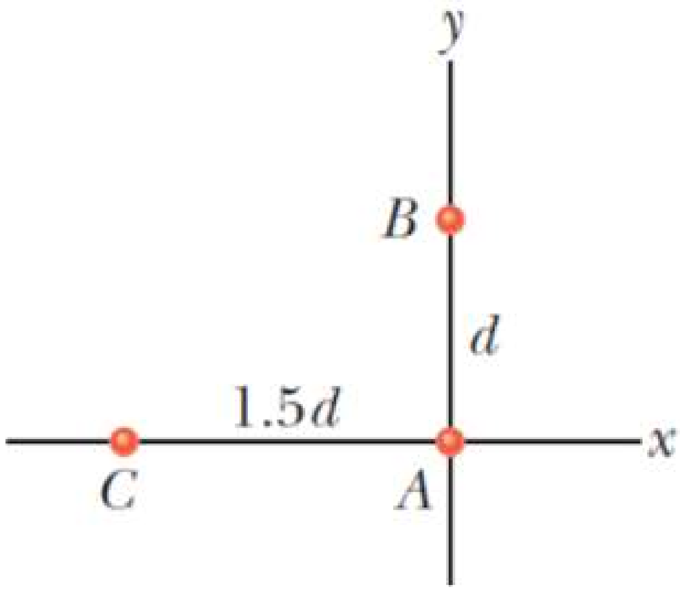
$y_0 = -1.5x_0, \quad x_0^2 + (-1.5x_0)^2 = 1.664$

$x_0^2 = 0.512, \quad x_0 = 0.716$

$y_0 = -1.5(0.716) = -1.07$

Following previous problem, find the potential energy of the system with A, B, C 3 particles in their places. U_g=_____ (01小題)

U_g=_____ [m,d,G]



24: ANS:=-G*m2/d*7.328**

$$U = U_{AB} + U_{BC} + U_{CA}$$

$$= \frac{-G(2m^2)}{d} + \frac{-G(6m^2)}{1.803d} + \frac{G(3m^2)}{1.5d}$$

$$= -\frac{Gm^2}{d}(2 + 3.328 + 2)$$

$$= -\frac{Gm^2}{d}(7.328)$$

Problem 6

A solid uniform sphere has a mass of 1.0×10^4 kg and a radius of 1.0 m. What is the magnitude of the gravity located at a distance of (a) 1.5 m and (b) 0.50 m from the center of the sphere. (02小題)

(a)for $r = 1.5$ m, g=______ m/s²

25: **ANS:=3E-7**

(b)for $r = 0.5$ m, g=______ m/s²

26: **ANS:=3.35E-7**

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4\,\text{kg}}{\frac{4}{3}\pi(1.0\text{ m})^3} = 2.4\times10^3\,\text{kg/m}^3.$$

$$\left|F_{\text{on } m}\right| = \frac{GmM_{\text{total}}}{r^2} = m\left(3.0\times10^{-7}\,\text{N/kg}\right).$$

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3\ \text{kg}.$$

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

Following previous problem, Write a general expression for the magnitude of the gravity at a distance $r \leq 1.0$ 1.0 m (inside the sphere). (01小題)

inside the sphere, $g(r)$=_______ [r]

27: **ANS:=6.7E-7*r**

PART.2: Consider a spherical asteroid whose radius is R and whose gravitational acceleration at the surface is a. (a)What is the escape speed on its surface? (b) With what speed will an object hit the asteroid if it is dropped from h above the surface?

(b)escape speed, v_{esc}=________ [a,R]

28: **ANS:=sqrt(2*a*R)**

(c)the object hit the surface of the asteroid with speed v=______ [a,R,h]

29: **ANS:=sqrt(2*a*R-(2*a*R**2)/(R+h))**

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

$$GM/R = a_g R,$$

$$-a_g R + \tfrac{1}{2}v^2 = 0.$$

$$v = \sqrt{2a_g R}$$

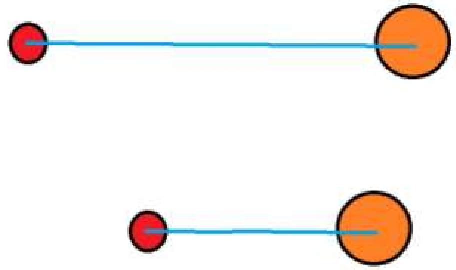
$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}}$$

Problem 7

Two balls of mass 10 kg and 40 kg are initially at rest and separated by a distance 1 km. (a)Find the gravitational force between them. Owing to this attractive force they begin to move toward each other. Find the velocity of each ball when their distance becomes 500 m. (03小題)



(a)|force|=_____ N

30: **ANS:=2.668E-11**

(b)the speed of 10 kg ball=______ m/s

31: **ANS:=2.066E-6**

(c)the speed of 40 kg ball=______ m/s

32: **ANS:=5.165E-7**

$$F = \frac{-6.67\times10^{-11}\times10\times40}{(10^3)^2} = 2.668\times10^{-14}\ N$$

$$E = U_i = \frac{-6.67\times10^{-11}\times10\times40}{10^3} = -2.668\times10^{-11}\ J$$

$$\Delta K = E - U = 2.668\times10^{-11}$$

動量守恆, $m\vec{v}_1 + m\vec{v}_4 = 0$,

$$10V_1 = 40V_4 \Rightarrow V_1 = 4V_4$$

$$\frac{1}{2}(10)V_1^2 + \frac{1}{2}(40)V_4^2 = \Delta K = 2.668\times10^{-11}$$

$$\frac{1}{2}(160 + 40)V_4^2 = 2.668\times10^{-11}$$

$$V_4^2 = 2.668\times10^{-13}$$

$$V_4 = 5.165\times10^{-7}\ m/s$$

$$V_1 = 4V_4 = 2.066\times10^{-6}\ m/s$$

Problem 7

Answer the following questions. (02小題)

(a)Earth is of mass 6×10^{24} kg and radius 6.4×10^{6} m. Find the gravitational acceleration at the center. g=_____ m/s^2

33: **ANS:=0**

(b)Following (a), if we have object shot into space from the surface of Earth. If the initial speed is just the escape speed, what is its mechanical energy? E=_____ J

34: **ANS:=0**