GPN1-Q09

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Problem 1

In a simple harmonic motion an object takes 0.4 s to travel from one point of zero velocity to the next such point. The distance between those points is 0.6 m. Calculate the following quantities. (05小題)

$T = 0.4 \times 2 = 0.8$

$A = \frac{0.6}{2} = 0.3$

(a)angular frequency, ω=____ rad/s

01: **ANS:=7.854**

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85$

(b)the maximum speed, v_{max}=_____ m/s

02: **ANS:=2.356**

$v_x = \omega A = 7.85(0.3) = 2.356$

(c)the maximum acceleration, a_{max}=_____ m/s^2

03: **ANS:=18.50**

$a_x = \omega^2 A = \omega v_x =$ 03: **ANS:=18.50**

(d)If the motion is driven by a spring with spring constant $k = 200$ N/m. Calculate the mass of the object. m=_____ kg

04: **ANS:=3.242**

$\omega = \sqrt{\frac{k}{m}}$, $m = \frac{k}{\omega^2} = \frac{200}{(7.85)^2} = 3.24$

(e)If we double the mass of the object, the frequency=_____ Hz

05: **ANS:=0.884**

$\omega \propto \frac{1}{\sqrt{m}} \Rightarrow \omega' = \omega \cdot \frac{1}{\sqrt{2}} = 5.55$

$f' = \frac{\omega'}{2\pi} = 0.884$

Problem 2

Answer the following questions: (04小題)

$A=3$ cm

$v=\frac{1}{2}v_x \Rightarrow \frac{K}{K_x}=\frac{\left(\frac{1}{2}v_x\right)^2}{v_x^2}=\frac{1}{4}$

$K=\frac{1}{4}k_x=\frac{1}{4}E \Rightarrow U=\frac{3}{4}E=\frac{3}{4}U_x$

$\frac{\frac{1}{2}kx^2}{\frac{1}{2}kA^2}=\frac{3}{4} \Rightarrow x=\frac{\sqrt{3}}{2}A=\frac{\sqrt{3}}{2}\cdot 3=2.60$

(a)A particle executes simple harmonic motion with an amplitude of 3.00 cm. At what displacement(x) from the midpoint of its motion does its speed equal one half of its maximum speed? $|x|$=_____ cm

06: **ANS:=2.6**

(b)A simple pendulum has a period of 2.50 s. What is its length? (b) What would its period be on the Moon, where $g_{Moon} = 1.67$ m/s^2 ?
length=_____ m

$T=2\pi\sqrt{\frac{L}{g}}$, $2.5=2\pi\sqrt{\frac{L}{9.8}}$, $L=1.55$

07: **ANS:=1.55**

period on the Moon=_____ s

$T=2\pi\sqrt{\frac{1.55}{1.67}}=6.05$ (s)

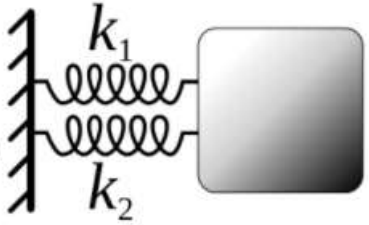
08: **ANS:=6.06**

If a simple pendulum with length 1.50 m makes 72.0 oscillations in 180 s, what is the acceleration of gravity at its location? g=_____ m/s^2

$T=\frac{180}{72}=2.5$, $g=\frac{4\pi^2L^2}{T^2}=\frac{4\pi^2(1.5)}{(2.5)^2}=9.47$

09: **ANS:=9.47**

As shown in the figure that an object of mass 2 kg is under SHM due to two parallel springs of $k_1 = k_2 = 100$ N/m. (01小題)



the period of the SHM=_____ s

$k_{eq}=k_1+k_2=200$

$T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{2}{200}}=0.1\pi$

10: **ANS:=0.6283**

Problem 3

一個進行簡諧運動的物體其位置對時間的函數，$x(t)$，可以下面的三角函數來表示：

$$x(t) = a\cos(bt + c)$$

請利用這個函數當中的三個參數(a,b,c)來表示這個簡諧震盪的：角頻率、振幅、週期、頻率、速度最大值、加速度的最大值。 (06小題)

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(1)角頻率=_____ [a,b,c]

11: **ANS:=b**

(2)振幅=_____ [a,b,c]

12: **ANS:=a**

(3)週期=_____ [a,b,c]

13: **ANS:=(2*pi)/b**

(4)頻率=_____ [a,b,c]

14: **ANS:=b/(2*pi)**

(5)速度最大值=_____ [a,b,c]

15: **ANS:=a*b**

(6)加速度的最大值=_____ [a,b,c]

$$X(t) = A\cos(\omega t + \phi)$$

$$v(t) = -\omega A\sin(\omega t + \phi)$$

$$a(t) = -\omega^2 A\cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega},\quad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$v_x = \omega A,\quad a_x = \omega^2 A$$

Problem 4

如果我們在行星的內部打通一個隧道穿過地心，將一個物體從隧道口的一端放下，這個物體(質量=u)將會因為行星的重力而進行落體運動，在隧道中的這個運動將會是一個簡諧運動。假設行星的質量是M，半徑是R，重力的宇宙常數G，(a)請計算行星的重力隨著物體與地心的距離x的函數關係；(b)請推導從隧道的一端運動到隧道的另一端口所需要的時間。 (02小題)

(a)$F_g(x)$=_____ [G,M,R,u,x] notice the sign of the force.

$F_g(x) = -ug\frac{x}{R} \rightarrow F_g(R) = -ug$ 表面重力

$= -u\left(\frac{GM}{R^2}\right)\frac{x}{R}$

$= -\frac{GMu}{R^3}x$

17: **ANS:=-u*G*M/R**3*x**

(b)t=_____ [G,M,R,u]

18: **ANS:=pi*sqrt(R**3/(G*M))**

Problem 4

(c)若將行星的質量$M = 8 \times 10^{24}$ kg和半徑$R = 8 \times 10^{6}$ m, $G = 6.67 \times 10^{-11}$代入你的(b)公式將可得到時間的數值。 (01小題)

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t=_____ min

19: **ANS:=51.3**

$k = \frac{GMu}{R^3}$, $T = 2\pi\sqrt{\frac{u}{k}} = 2\pi\sqrt{\frac{R^3}{GM}}$

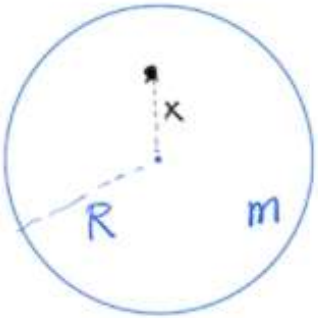
單趟：$\frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$

$\pi\sqrt{\frac{(8\times10^6)^3}{6.67\times10^{-11}(8\times10^{24})}} = 3077(s)$

$= 51.3(min)$

Problem 4 (公式題)

有一個半徑R質量m的圓盤鉛垂懸掛如圖所示，旋轉軸的支點與圓盤中心的距離是x，如果將這個圓盤微微的從千錘鉛垂下移動一個小角度再釋放，這個圓盤將會進行物理擺的週期運動。(a)請計算這個圓盤相對於旋轉軸的轉動慣量。這個圓盤進行簡諧運動的週期是x的函數，$T(x)$，(b)請計算要得到最小的週期，x=? 並且(c)計算這個最小的週期的公式T_{min}。 (03小題)



$1/2mR^2 + mx^2$

$R/\sqrt{2}$

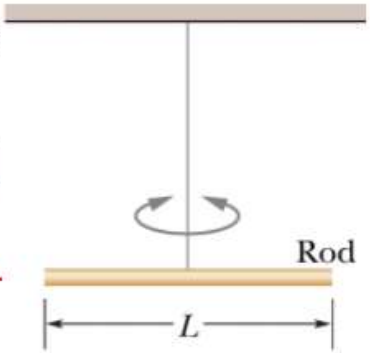
$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$$= 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + Md^2}{Mgd}}$$

$$= 2\pi\sqrt{\frac{R^2+2d^2}{2gd}}$$

Problem 5

如圖所示有一個質量0.5 kg，長度2 m的竿子，一個鋼繩從棒子的中心點將竿子懸掛起來，竿子可進行扭擺(torsion pendulum)的簡諧運動，週期為0.5 s，請計算這個扭擺的扭力常數。如果有一個物體附著在這個竿子的一端，則扭擺的週期增加為0.6 s，請計算這個物體的質量。 (03小題)



(a)扭力常數,κ=_____ N.m

20: **ANS:=26.32**

$I_r = \frac{1}{12}ML^2 = 0.1667 \quad T = 2\pi\sqrt{\frac{I_r}{\kappa}}$

(b)物體的質量=_____ kg

21: **ANS:=0.0733**

$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(\frac{1}{12})(0.5)(2)^2}{(0.5)^2} = 26.32$

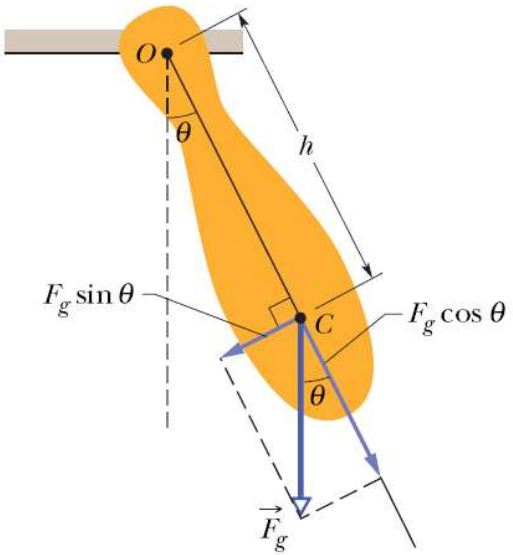
(c)A physical pendulum in the form of a planar body moves in simple harmonic motion with a frequency of 0.450 Hz. If the pendulum has a mass of 2.20 kg and the pivot is located 0.350 m from the center of mass, determine the moment of inertia of the pendulum. I=_____ kg.m^2

22: **ANS:=0.944**

$I = \kappa\left(\frac{T^2}{4\pi^2}\right) = 26.32\left(\frac{0.6^2}{4\pi^2}\right) = 0.240$

$I = I_r + m(1)^2 = 0.1667 + m = 0.240$

$m = 0.0733$



$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

$I = (mgh)\left(\frac{T}{2\pi}\right)^2$

$= (2.2)(9.8)(0.35)\left(\frac{1}{2\pi(0.45)}\right)^2$

$= 0.944$

Problem 5

A simple pendulum having a length of 2.23 m and a mass of 6.74 kg is given an initial speed of 2.06 m/s at its equilibrium position. Assume it undergoes simple harmonic motion and determine its (a) period, (b) total energy, and (c) maximum angular displacement. (03小題)

(a) period, T=____ s

23: **ANS:=3.02**

(b) total energy, E=_____ J

24: **ANS:=14.3**

(c) maximum angular displacement=_____ rad

25: **ANS:=0.444**

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.23}{9.8}} = 2.997\,(s)$$

$$K_x = E = \frac{1}{2}(6.74)(2.06)^2 = 14.30$$

$$v_x = 2.06 \Rightarrow \omega_x = \frac{v_x}{L} = \frac{2.06}{2.23} = 0.924\,(rad/s)$$

$$\theta = \theta_m \cos(\omega t + \phi), \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2.997} = 2.096$$

$$\omega = -\omega\theta_m \sin(\omega t + \phi)$$

$$\omega\theta_m = \omega_x, \quad \theta_m = \frac{\omega_x}{\omega} = \frac{0.924}{2.096} = 0.441$$

Problem 6

When the displacement in SHM is $x = 1/4x_m$, x_m is the amplitude, what fraction of the total energy,E is (a) kinetic energy,K and (b) potential energy,U? (c)When the kinetic energy is twice the potential energy, please calculate the distance between the simple harmonic oscillator and the equilibrium point. (03小題)

(a)$\frac{K}{E}$=_____

26: **ANS:=15/16**

(b)$\frac{U}{E}$=_____

27: **ANS:=1/16**

(a)x=_____ x_m

28: **ANS:=1/sqrt(3)**

$X = \frac{1}{4}X_m$

$U = \frac{1}{2}kX^2$

$= \frac{1}{2}k\left(\frac{1}{4}X_m\right)^2$

$= \frac{1}{16}\left(\frac{1}{2}kX_m^2\right)$

$= \frac{1}{16}E$

$K = E - U = \frac{15}{16}E$

$\frac{K}{E} = \frac{15}{16} = 0.9375$

$\frac{U}{E} = \frac{1}{16} = 0.0625$

$K = 2U = \frac{2}{3}E$

$U = \frac{1}{3}E$

$\frac{1}{2}kX^2 = \frac{1}{3}\left(\frac{1}{2}kX_m\right)$

$X = \frac{1}{\sqrt{3}}X_m$

Problem 6

A block rides on a piston that is moving vertically with simple harmonic motion. (a) If the SHM has period 6 s, at what amplitude of motion will the block and piston separate? (b) If the piston has an amplitude of 1.2 cm, what is the maximum frequency for which the block and piston will be in contact continuously? (02小題)

(a)A=____ m

29: **ANS:=8.94**

(b)f=_____ Hz

30: **ANS:=4.55**

$g = a_x = \omega^2 A$,

$A = \frac{g}{\omega^2} = \frac{g}{\left(\frac{2\pi}{T}\right)^2} = \frac{gT^2}{4\pi^2}$

$= \frac{9.8(6)^2}{4\pi^2} = 8.94$

$\omega^2 = \frac{g}{A} = \frac{9.8}{0.012} = 816.7$

$\omega = 28.58$

$f = \frac{\omega}{2\pi} = \frac{28.58}{2\pi} = 4.55$