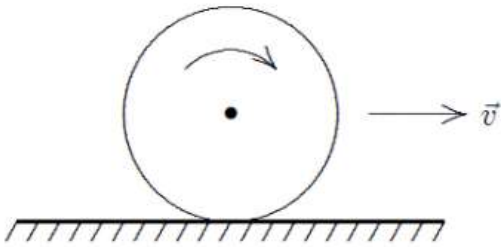
GPN1-L07

Problem 1

A wheel of radius 0.5 m rolls without sliding on a horizontal surface as shown. Starting from rest, the wheel moves with constant angular acceleration 6 rad/s^2. The distance traveled by the center of the wheel from t = 0 to t = 3 s. (01小題)



The distance=_____ m

$S = R\theta$

$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(6)(3)^2 = 27$ rad

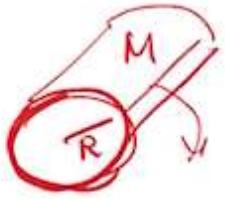
$S = 0.5(27) = 13.5$ m

01: **ANS:=13.5**

A thin-walled hollow tube rolls without sliding along the floor. Find the ratio of its translational kinetic energy to its rotational kinetic energy (about an axis through its center of mass). (01小題)

the ratio=_____

thin-walled hollow tube :



$I = MR^2$

$v = R\omega$

$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$= \frac{1}{2}MR^2\omega^2 + \frac{1}{2}MR^2\omega^2$

$\frac{K_{rot}}{K_{tr}} = 1$

02: **ANS:=1**

Problem 2

Answer the following questions: (03小題)

(a)The coefficient of static friction between a certain cylinder and a horizontal floor is 0.40. If the rotational inertia of the cylinder about its symmetry axis is given by $I = (1/2)MR^2$, then the magnitude of the maximum acceleration the cylinder can have without sliding=_____ m/s^2

$a = R\alpha$ $\quad Rf_s = I\alpha$ $\quad \alpha = \frac{2\mu_s g}{R}$, $\quad a = R\alpha = 2\mu_s g$

$f_s = \mu_s Mg$ $\quad \mu_s Mg R = \frac{1}{2}MR^2\alpha$ $\quad = 2(0.4)(9.8)$

$= 7.84$

03: **ANS:=7.84**

(b)Two identical disks, with rotational inertia $I = 1/2MR^2$, roll without sliding across a horizontal floor with the same speed and then up inclines. Disk A rolls up its incline without sliding. On the other hand, disk B rolls up a frictionless incline. Otherwise the inclines are identical. Disk A reaches a height 12 cm above the floor before rolling down again. Disk B reaches a height above the floor of h, h=_____ cm

04: **ANS:=8**

(c)Two identical hoops roll without sliding across a horizontal floor with the same speed and then up inclines. hoop A rolls up its incline without sliding. On the other hand, hoop B rolls up a frictionless incline. 2 inclines are of the same incline angle and length. Disk A reaches a height H above the floor before rolling down again. Find the height above the floor disk B reaches, H'=_____ H.

05: **ANS:=0.5**

$K_{tr} = \frac{1}{2}Mv^2$, $K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$, $v = R\omega$, $I = \frac{1}{2}MR^2$

$K_{rot} = \frac{1}{2}K_{tr} = K_0$

$K = K_{rot} + K_{tr} = 3K_0$

$3K_0 = Mg(12)$, $K_{rot} = K_0 = 4Mg$

$K_{rot} + Mgh = Mg(12)$

$4Mg + Mgh = 12Mg$, $h = 8$ (cm)

$I = MR^2 \Rightarrow K_{rot} = K_{tr} = K_0$

$K = 2K_0 = MgH$, $K_0 = \frac{1}{2}MgH$

$MgH' + K_0 = MgH \Rightarrow MgH' = \frac{1}{2}MgH$

$\Rightarrow H' = \frac{1}{2}H$

L07 P03為何題目中給的是動摩擦係數呢？這題跟動摩擦力有關係嗎

Problem 3

A cylinder of radius R = 6.0 cm is on a rough horizontal surface. The coefficient of kinetic friction between the cylinder and the surface is 0.30 and the rotational inertia for rotation about the axis is given by $MR^2/2$, where M is its mass. Initially it is not rotating but its center of mass has a speed of 7.0 m/s. After 2.0 s, find the speed of its center of mass and its angular velocity about its center of mass. (02小題)

(a)the speed of its center of mass =_____ m/s

06: **ANS:=1.12**

(b)angular velocity about its center of mass=_____ rad/s

07: **ANS:=196**

$f_k = \mu_k mg$

$= (0.3)(M)(9.8)$

$a = \frac{f_k}{m} = (0.3)(9.8) = 2.94$

$v = v_0 + (-2.94)(2) = 1.12$

$\alpha = \frac{Rf_k}{I} = \frac{(0.06)(0.3)(9.8)M}{\frac{1}{2}M(0.06)^2} = 98$

$\omega = \omega_0 + \alpha t = 0 + (98)(2) = 196$

Problem 4

A solid wheel with mass M, radius R, rolls without sliding on a horizontal surface. The wheel can be treated as a disk. A horizontal force is applied to the axle and the center of mass has an acceleration a. Find the magnitudes of the applied force F and the frictional force f of the surface. (02小題)

(a)F=_____ [M,R,a]

08: **ANS:=3/2*M*a**

(b)f=_____ [M,R,a]

09: **ANS:=1/2*M*a**

$$F - f = Ma$$

$$Rf = I\alpha = \frac{1}{2}MR^2\alpha$$

$$f = \frac{1}{2}Ma$$

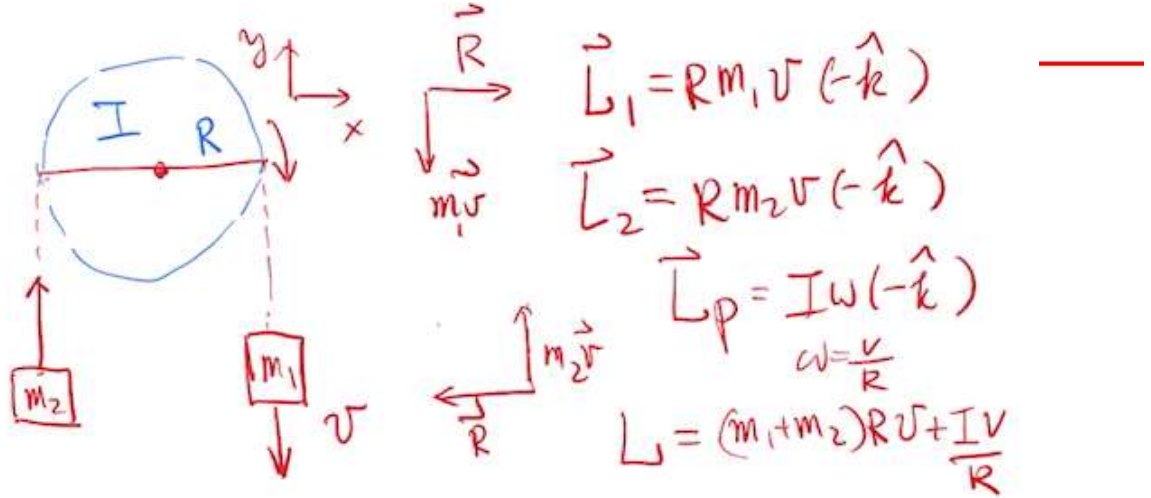
$$F = Ma + \frac{1}{2}Ma = \frac{3}{2}Ma$$

Problem 4

A pulley with radius R and rotational inertia I is free to rotate on a horizontal fixed axis through its center. A string passes over the pulley. A block of mass m1 is attached to one end and a block of mass m2 is attached to the other. At one time the block with mass m1 is moving downward with speed v. If the string does not slip on the pulley, the magnitude of the total angular momentum, about the pulley center, of the blocks and pulley, considered as a system, is given by:__

L=_____ [m1,m2,R,I,v]

10: **ANS:=(m1+m2)*v*R+I*v/R**



$$\vec{L}_1 = Rm_1v(-\hat{k})$$

$$\vec{L}_2 = Rm_2v(-\hat{k})$$

$$\vec{L}_p = I\omega(-\hat{k})$$

$$\omega = \frac{v}{R}$$

$$L = (m_1+m_2)Rv + \frac{Iv}{R}$$

Problem 5

Answer the following questions: (04小題)

(a)As a 2.0-kg block travels around a 0.50-m radius circle it has an angular speed of 12 rad/s. The circle is parallel to the xy plane and is centered on the z axis, 0.75 m from the origin. The magnitude of its angular momentum around the origin =_____ kg.m/s^2

11: **ANS:=11**

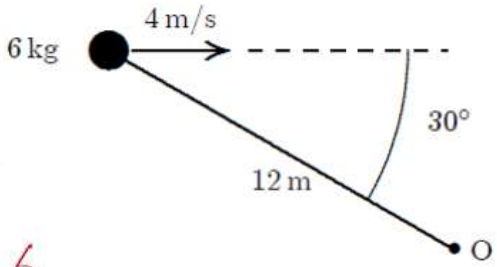
(b)Following (a), the The z component of the angular momentum around the origin=_____ kg.m/s^2

12: **ANS:=6**

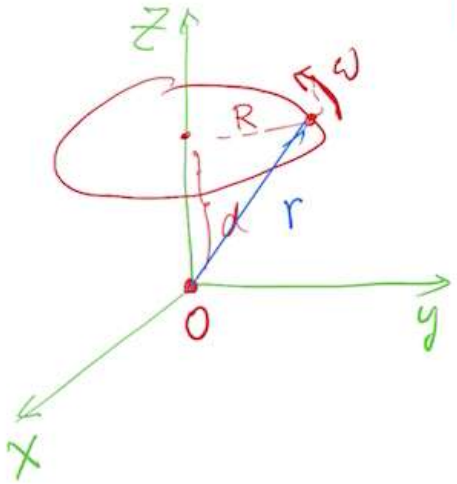
(c)Following (a,b), the The component on the xy plane of the angular momentum around the origin=_____ kg.m/s^2

13: **ANS:=9**

(b)A 6.0-kg particle moves to the right at 4.0 m/s as shown. The magnitude of its angular momentum about the point O, L =_____ kg.m/s^2



14: **ANS:=144**



$\vec{L} = \vec{r} \times m\vec{v}$

$= (0, R, d) \times (-mv, 0, 0)$

$= (mvR)\hat{k} - mvd\hat{\imath}$

$|\vec{L}| = \sqrt{(mvR)^2 + (mvd)^2}$

$= 2(0.5)(12)\sqrt{0.5^2 + 0.75^2}$

$= 10.82$

$L_z = mvR = 2(0.5)^2(12) = 6$

$L_{xy} = mvd = 2(0.5)(12)(0.75) = 9$

$L = |\vec{r} \times \vec{p}| = (12)(6\times 4)\sin 30^\circ = 144$

Problem 6

Answer the following questions: (03小題)

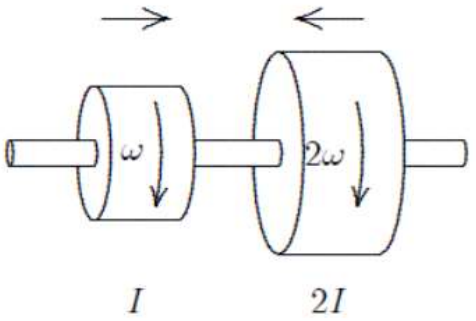
(a)An ice skater with rotational inertia I is spinning with angular speed ω. She pulls her arms in, thereby increasing her angular speed to 4ω. Her rotational inertia I'=_____ I

15: **ANS:=0.25**

(b)A uniform sphere of radius R rotates about a diameter with an angular momentum of magnitude L. Under the action of internal forces the sphere collapses to a uniform sphere of radius $R/2$. The magnitude of its new angular momentum L'=_____ L

16: **ANS:=1.0**

(c) Two disks are mounted on low-friction bearings on a common shaft. The first disc has rotational inertia I and is spinning with angular velocity ω. The second disc has rotational inertia $2I$ and is spinning in the same direction as the first disc with angular velocity 2ω as shown. The two disks are slowly forced toward each other along the shaft until they couple and have a final common angular velocity ω'=_____ ω



17: **ANS:=5/3**

$I\omega = I'(4\omega)$

$I' = \frac{1}{4}\omega$

$L' = L$

$I\omega + 2I(2\omega) = 3I\omega'$

$\omega' = \frac{5}{3}\omega$

Problem 7

A particle moves through an xyz coordinate system while a force acts on the particle. When the particle has the position vector $\vec{r} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ the force is $\vec{F} = F_x\hat{i} + 7\hat{j} - 6\hat{k}$ and the corresponding torque about the origin is $\vec{\tau} = 4\hat{i} + 2\hat{j} - 1\hat{k}$. All quantities are in SIunits. Determine F_x. (01小題)

F_x=_____ N

18: **ANS:=-5**

$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ F_x & 7 & -6 \end{vmatrix} = (4, 2F_x+12, 14+3F_x)$

$= (4, 2, -1)$

$2F_x + 12 = 2$, $F_x = -5$

At the instant the displacement of a 2.00 kg object relative to the origin is $\vec{d} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ its velocity is $\vec{v} = -6\hat{i} + 3\hat{j} + 3\hat{k}$ and it is subject to a force $\vec{F} = 6\hat{i} - 8\hat{j} + 4\hat{k}$. Find the following quantities. (07小題)

(a) the acceleration of the object. a_x=_____ m/s^2

19: **ANS:=3**

a_y=_____ m/s^2

20: **ANS:=-4**

$\vec{a} = \vec{F}/m = \frac{(6, -8, 4)}{2} = (3, -4, 2)$

(b) the angular momentum of the object about the origin, L_x=_____ kg.m^2/s

21: **ANS:=42**

L_z=_____ kg.m^2/s

22: **ANS:=60**

$\vec{L} = \vec{r} \times \vec{p} = (2, 4, -3) \times 2(-6, 3, 3)$

$= (42, -24, 60)$

(c) the torque about the origin acting on the object, τ_y=_____ Nm

23: **ANS:=-26**

τ_z=_____ Nm

24: **ANS:=-40**

$\vec{\tau} = \vec{r} \times \vec{F} = (2, 4, -3) \times (6, -8, 4)$

$= (-8, -26, -40)$

$\cos\theta = \frac{\vec{v} \cdot \vec{F}}{v\,F} = \_\_\_\_ = -0.606$

$\theta = 127^\circ$

(d)the angle between the velocity of the object and the force acting on the object=_____ degree

25: **ANS:=127**