GPN1-L13

Problem 1

Answer the following questions: $(05/\sqrt{5})$

(a)Gold has a molar mass of 197 g/mol. (a1) How many moles(n) of gold are in a 2.50 g sample of pure gold? (a2) How many atoms (N) are in the sample?

 $(a1)n=$

(a2) $N=$

 $n = M_{\text{sam}}/M = 2.5/197 = 0.0127$ mol.

$01: ANS := 0.0127$

$$
N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}
$$

02: ANS:=7.64E21

(b)Find the mass(m) in kilograms of 7.50×10^{24} atoms of arsenic, which has a molar mass of 74.9 g/mol. m = kg

 (7.50×10^{24}) $(74.9 \times 10^{-3} \text{ kg/mol})/(6.02 \times 10^{23} \text{ mol}^{-1}) = 0.933 \text{ kg}.$ $03: ANS := 0.933$

(c)The best laboratory vacuum has a pressure of about 1.00×10^{-18} atm, or 1.01×10^{-13} Pa. How many gas molecules are there per cubic centimeter in such a vacuum at 293 K? $N=$

04: ANS: = 25

(d)Compute (a) the number of moles(n) of molecules in 1.00 cm³ of an ideal gas at a pressure of 100 Pa and a temperature of 220 K, $n=$

$05: ANS := 5.47E-8$

$$
n = \frac{pV}{RT} = \frac{(100\,\text{Pa})(1.0 \times 10^{-6}\,\text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(220\,\text{K})} = 5.47 \times 10^{-8}\,\text{mol}.
$$

$$
V = 1.0 \times 10^{-6}
$$
 m³, $p = 1.01 \times 10^{-13}$ Pa, and $T = 293$ K,

$$
n = \frac{pV}{RT} = \frac{(1.01 \times 10^{-13} \text{ Pa})(1.0 \times 10^{-6} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 4.1 \times 10^{-23} \text{ mole}
$$

 $N = nN_A = 25$ molecules.

A quantity of ideal gas at 10.0°C and 100 kPa occupies a volume of 2.50 m³. (a) How many moles, n, of the gas are present? (b) If the pressure is now raised to 300 kPa and the temperature is raised to 30.0° C, how much volume *V* does the gas occupy? Assume no leaks. $(02/\sqrt{5})$

 $(a)n=$

 $06: ANS := 106$

 $(b)V = m^3$

$$
n = \frac{pV}{RT} = \frac{(100 \times 10^3 \text{ Pa})(2.50 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(283 \text{ K})} = 106 \text{mol}
$$

 $07: ANS: = 0.892$

$$
\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad V_f = V_i \left(\frac{p_i}{p_f}\right) \left(\frac{T_f}{T_i}\right) = (2.50 \,\text{m}^3) \left(\frac{100 \,\text{kPa}}{300 \,\text{kPa}}\right) \left(\frac{303 \,\text{K}}{283 \,\text{K}}\right) = 0.892 \,\text{m}^3
$$

Problem 2

Suppose 1.80 mol of an ideal gas is taken from a volume of 3.00 $m³$ to a volume of 1.50 $m³$ via an isothermal compression at 30°C. (a) How much energy is transferred as heat during the compression, and (b) is the transfer to or from the gas? $(02/1)$ 題)

(a) the magnitude of energy transferred as heat = 1

08: ANS:=3.14E3

 $(b)1$ =the heat transfer to the gas; 2=the heat transfer from the gas. Ans.=

09: ANS:=2

$$
Q = W.
$$

$$
W = nRT \ln \frac{V_f}{V_i} \quad T = (273 + 30.0) \text{K}
$$

$$
Q = -3.14 \times 10^3 \text{ J, or } |Q| = 3.14 \times 10^3 \text{ J.}
$$

That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

An air bubble of volume 20 cm3 is at the bottom of a lake 40 m deep, where the temperature is 4.0° C. The bubble rises to the surface, which is at a temperature of 20°C. Take the temperature of the bubble's air to be the same as that of the surrounding water. Just as the bubble reaches the surface, what is its volume? (01小題)

its volume at the surface, $V_2 =$ _______ cm³

$10: ANS := 100$

We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1V_1 = nRT_1$, the number of moles of gas in the bubble is

 $n = p_1 V_1/RT_1 = (p_0 + \rho g d) V_1/RT_1$

where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is V_2 = nRT_2/p_0 . We substitute for *n* to obtain

$$
V_2 = \frac{T_2}{T_1} \frac{p_0 + \rho g d}{p_0} V_1
$$

= $\left(\frac{293 \text{ K}}{277 \text{ K}}\right) \left(\frac{1.013 \times 10^5 \text{ Pa} + (0.998 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(40 \text{ m})}{1.013 \times 10^5 \text{ Pa}}\right) (20 \text{ cm}^3)$
= $1.0 \times 10^2 \text{ cm}^3$.

Answer the following questions: $(04/\sqrt{5})$

(a) The lowest possible temperature in outer space is 2.7 K. The rms speed of hydrogen molecules at this temperature, V_{rms} $=$ m/s

$11: ANS := 180$

$$
v_{\rm rms} = \sqrt{\frac{3 (8.31 \text{ J/mol} \cdot \text{K})(2.7 \text{ K})}{2.02 \times 10^{-3} \text{ kg/mol}}} = 1.8 \times 10^2 \text{ m/s}.
$$

(b) The temperature and pressure in the Sun's atmosphere are 2.00×10^6 K and 0.0300 Pa. Calculate the rms speed of free electrons (mass 9.11×10^{-31} kg) there, assuming they are an ideal gas. $V_{rms} =$ _____ m/s $\sqrt{3RT}$ $\sqrt{3(kN_A)T}$ $\sqrt{3kT}$

12: ANS: = 9.53E6
$$
k = 1.38 \times 10^{-23}
$$
 J/K $T = 2.00 \times 10^6$ K, $v_{\text{rms}} = 9.53 \times 10^6$ m/s. $\sqrt{v_{\text{rms}} - \sqrt{M} - \sqrt{(mN_A)} - \sqrt{M}}.$

(c)A beam of hydrogen molecules (H^2) is directed toward a wall, at an angle of 55° with the normal to the wall. Each molecule in the beam has a speed of 1000 m/s and a mass of 3.3×10^{-24} g. The beam strikes the wall over an area of 2.0 cm², at the rate of 10^{23} molecules per second. What is the beam's pressure on the wall? $P =$ ______ Pa

13: ANS:=1.9E3

(d)What is the average translational kinetic energy of nitrogen molecules at 1600 K? $K =$ ______ J

 14 : ANS: = 3.31E-20

$$
p = \frac{2}{A} \left(\frac{N}{\Delta t}\right) mv \cos \theta
$$

= $\left(\frac{2}{2.0 \times 10^{-4} \text{ m}^2}\right) (1.0 \times 10^{23} \text{ s}^{-1}) (3.3 \times 10^{-27} \text{ kg}) (1.0 \times 10^3 \text{ m/s}) \cos 55^\circ$
= $1.9 \times 10^3 \text{ Pa.}$

$$
K_{\text{avg}} = \frac{3}{2}kT \qquad k = 1.38 \times 10^{-23} \text{ J/K}
$$

$$
K_{\text{avg}} = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K}) (1600 \text{ K}) = 3.31 \times 10^{-20} \text{ J}
$$

Ten particles are moving with the following speeds: four at 200 m/s, two at 500 m/s, and four at 600 m/s. Calculate their (a) average and (b) rms speeds. $(02/\sqrt{5})$

(a) average speed=
\n
$$
m/s
$$
\n
$$
v_{avg} = \frac{1}{N} \sum_{i=1}^{N} v_i = \frac{1}{10} [4(200 \text{ m/s}) + 2(500 \text{ m/s}) + 4(600 \text{ m/s})] = 420 \text{ m/s}.
$$
\n(b) rms speed=
\n
$$
m/s
$$
\n
$$
v_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10} [4(200 \text{ m/s})^2 + 2(500 \text{ m/s})^2 + 4(600 \text{ m/s})^2] } = 458 \text{ m/s}
$$

Problem 5

The figure gives the probability distribution for nitrogen gas. The scale of the horizontal axis is set by $vs = 1200$ m/s. What are the (a) gas temperature and (b) rms speed of the molecules? $(02/1)$ 題)

(a)the gas temperature, $T =$ K

$17: ANS = 270$

(b) rms speed of the molecules=_______ m/s

 $18: ANS := 490$

$$
v_p = 400 \text{ m/s}.
$$
 $M = 28 \text{ g/mol}$
 $T = \frac{1}{2} M v_p^2 / R = 2.7 \times 10^2 \text{ K}.$ $v_{\text{rms}} = \sqrt{3/2} v_p = 4.9 \times 10^2 \text{ m/s}.$

The figure shows a hypothetical speed distribution for a sample of N gas particles (note that $P(v) = 0$ for speed $v > 2v_0$). What are the values of (a) $a_v 0$, (b) v_{avg}/v_0 , $\sum_{n=1}^{\infty}$ and (c) v_{rms}/v_0 ? (d) What fraction of the particles has a speed between 1.5 v_0 and $2.0v_0$? (04小題)

$$
\underline{19:} \text{ANS:} = \underline{2/3}
$$

 $20: ANS:=1.22$

 $21: ANS := 1.31$

(d)the fraction between $1.5v_0$ and $2.0v_0 =$

 $22: ANS: = 1/3$

$$
\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0,
$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

$$
v_{avg} = \int \psi P(v) dv.
$$

$$
\frac{a}{v_0} \int_0^{v_0} v^2 dv = \frac{a}{3v_0} v_0^3 = \frac{av_0^2}{3} = \frac{2}{9} v_0,
$$

$$
P(v) = a
$$
 in the rectangular portion,
$$
a \int_{v_0}^{2v_0} v dv = \frac{a}{2} (4v_0^2 - v_0^2) = \frac{3a}{2} v_0^2 = v_0.
$$

$$
v_{avg} = \frac{2}{9} v_0 + v_0 = 1.22 v_0 \implies \frac{v_{avg}}{v_0} = 1.22.
$$

$$
v_{\text{rms}}^2 = \int v^2 P(v) dv.
$$

\n
$$
\frac{a}{v_0} \int_0^{v_0} v^3 dv = \frac{a}{4v_0} v_0^4 = \frac{1}{6} v_0^2.
$$

\n
$$
a \int_{v_0}^{2v_0} v^2 dv = \frac{a}{3} (8v_0^3 - v_0^3) = \frac{7a}{3} v_0^3 = \frac{14}{9} v_0^2.
$$

\n
$$
v_{\text{rms}} = \sqrt{\frac{1}{6} v_0^2 + \frac{14}{9} v_0^2} = 1.31 v_0 \frac{v_{\text{rms}}}{v_0} = 1.31.
$$

\n
$$
N \int_{1.5v_0}^{2v_0} P(v) dv = Na(2.0v_0 - 1.5v_0)
$$

\n
$$
= 0.5N av_0 = N/3,
$$

\nthe fraction of particles = 1/3