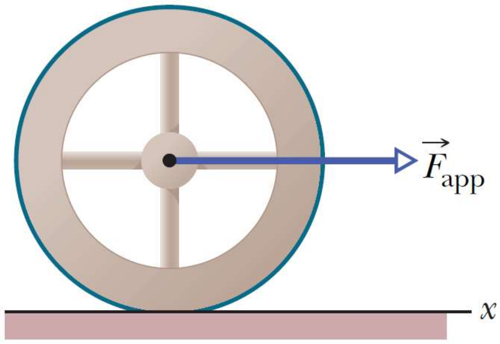
GPN1-LS07

Problem 1

In the figure, constant horizontal force of magnitude 10 N is applied to a wheel of mass 10 kg and radius 0.30 m. The wheel rolls smoothly on the horizontal surface, and the acceleration of its center of mass has magnitude 0.60 m/s^2. (a)What is the frictional force on the wheel? (b) What is the rotational inertia of the wheel about the rotation axis through its center of mass?



在圖中，大小為10 N的恆定水平力施加到質量為10 kg且半徑為0.30 m的車輪上。 車輪在水平面上平穩滾動，其質心加速度為0.60 m/s^2。(a)車輪上的摩擦力是多少？ (b)車輪通過其質心繞旋轉軸的旋轉慣量是多少？ (02小題)

(a)magnitude of the frictional force on the wheel=_____ N

01: **ANS:=<u>4</u>**

(b)the rotational inertia of the wheel=______ kg.m^2

02: **ANS:=<u>0.6</u>**

Solution:

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\,\text{N} - (10\,\text{kg})(0.60\ \text{m/s}^2) = 4.0\ \text{N}.$$

$$\vec{f}_s = (-4.0\ \text{N})\hat{\text{i}}$$

the acceleration of its center of mass has magnitude $0.60\ \text{m/s}^2$.

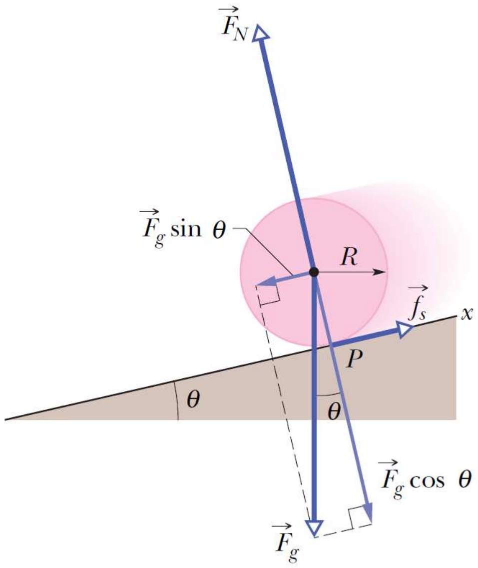
$|\alpha| = |a_{com}| / R = 2.0\ \text{rad/s}^2$, $\quad R = 0.30\ \text{m}$,

The only force not directed towards (or away from) the center of mass is $\vec{f}_s$,

$$|\tau| = I|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I\left(2.0\,\text{rad/s}^2\right)$$

Problem 2

A uniform ball, of mass $M = 6.00$ kg and radius R, rolls smoothly from rest down a ramp at angle $\theta = 30.0^\circ$ (see figure). (a) The ball descends a vertical height $h = 1.20$ m to reach the bottom of the ramp. What is its speed at the bottom? (b) What are the magnitude and (c)direction of the frictional force on the ball as it rolls down the ramp? (03小題)



(a)speed at bottom=_______ m/s

03: **ANS:=4.10**

(b)magnitude of the firctional force=_______ N

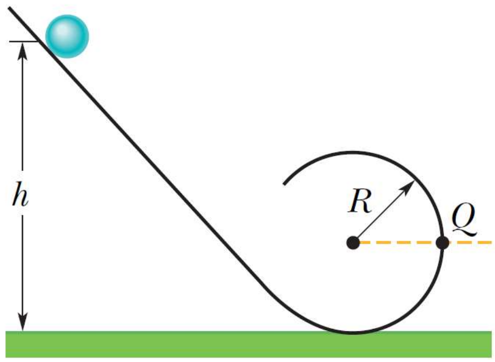
04: **ANS:=8.4**

(c)the direction of friction is in: 1=+x; 2=-x; 3=+y; 4=-y

05: **ANS:=1**

Problem 3

In the figure, a solid brass ball of mass 0.280 g will roll smoothly along a loop-the-loop track when released from rest along the straight section. The circular loop has radius R = 14.0 cm, and the ball has radius $r \ll R$. (a) What is h if the ball is on the verge of leaving the track when it reaches the top of the loop? If the ball is released at height h = 6 R, what are the (b) magnitude of the horizontal force component acting on the ball at point Q?



在圖中，質量為0.280 g的實心黃銅球在沿直線部分從靜止狀態釋放時，將沿著環回軌道平滑滾動。 圓環的半徑為$R = 14.0$ cm，球的半徑為$r \ll R$。 （a）如果球快要到達圈頂時就快要離開賽道了，那h是多少？ 如果在高度$h = 6R$處釋放球，在點Q上作用在球上的(b)水平力分量的大小是多少？ (02小題)

(a)h=_____ cm

06: **ANS:=<u>37.8</u>**

(b) N=______ N

07: **ANS:=<u>1.96E-2</u>**

Solution:

8. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass m in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction (+y downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R$. With R = 14.0 cm , we have h = (2.7)(14.0 cm) = 37.8 cm.

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \frac{7}{10}v_{\text{com}}^2 + gR$$

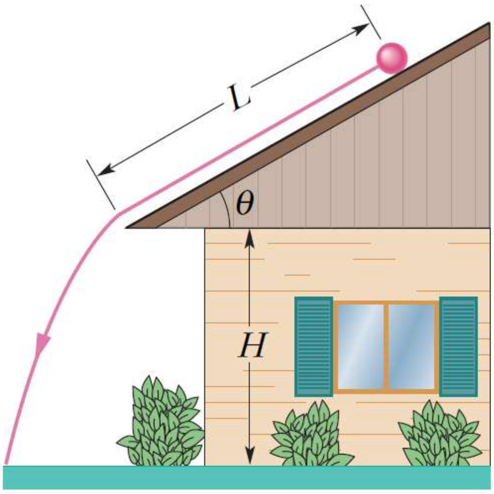
which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q leads to

$$N = m\frac{v_{\text{com}}^2}{R-r} = m\frac{50gR}{7(R-r)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{7} = 1.96\times10^{-2}\ \text{N}.$$

Problem 4



In the figure, a solid cylinder of radius 10 cm and mass 12 kg starts from rest and rolls without slipping a distance L = 6.0 m down a roof that is inclined at the angle $\theta = 30^\circ$.

(a)What is the angular speed of the cylinder about its center as it leaves the roof? (b) The roof's edge is at height H = 5.0 m. How far horizontally from the roof's edge does the cylinder hit the level ground?

在圖中，半徑為10 cm且質量為12 kg的實心圓柱體從靜止狀態開始滾動，並且沿傾斜角度為$\theta = 30^\circ$的車頂向下滑動距離$L = 6.0$ m。 (a)圓柱離開屋頂時繞其中心的角速度是多少？ (b) 屋頂邊緣的高度為$H = 5.0$ m。 圓柱體與屋頂邊緣水平相距多遠？ (02小題)

(a)the angular speed, ω=____ rad./s

08: **ANS:=<u>63</u>**

(b)the distance from the roof's edge=_____ m

09: **ANS:=<u>4</u>**

Solution:

$h = 6.0 \sin 30° = 3.0 \text{ m}$

$U_i = Mgh$

$K_f = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 .$

$v = R\omega \qquad I = \frac{1}{2} MR^2$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2 .$$

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10 \text{ m}}\sqrt{\frac{4}{3}(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 63 \text{ rad/s}.$$

$v_0 = R\omega = 6.3 \text{ m/s},$

$v_{0x} = v_0 \cos 30° = 5.4 \text{ m/s}$

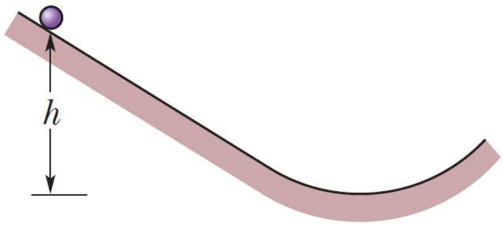
$v_{0y} = v_0 \sin 30° = 3.1 \text{ m/s}.$

$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$

$$y = H = 5.0 \text{ m} \qquad t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74 \text{ s}.$$

$x = (5.4 \text{ m/s})(0.74 \text{ s}) = 4.0 \text{ m}.$

Problem 5



In the figure, rolls smoothly from rest down a ramp and onto a circular loop of radius 0.48 m. The initial height of the ball is h = 0.36 m. At the loop bottom, the magnitude of the normal force on the ball is 2Mg. The ball consists of an outer spherical shell (of a certain uniform density) that is glued to a central sphere (of a different uniform density). The rotational inertia of the ball can be expressed in the general form $I = \beta MR^2$, but β is not 0.4 as it is for a ball of uniform density. Determine β.

在圖中，從靜止的地方順著坡道順滑滾動到半徑為0.48 m的圓環上。球的初始高度為$h = 0.36$ m。在環的底部，作用在球上的法向力的大小為$2Mg$。球由一個外部球殼（具有一定的均勻密度）組成，該球殼膠合到一個中心球(具有不同的均勻密度)上。 球的旋轉慣性可以用形式$I = \beta MR^2$來表達，但是β不是0.4。計算β。 (01小題)

β=____

10: **ANS:=<u>0.5</u>**

Solution:

$$F_N - Mg = Mv^2/r \qquad F_N = 2Mg)$$

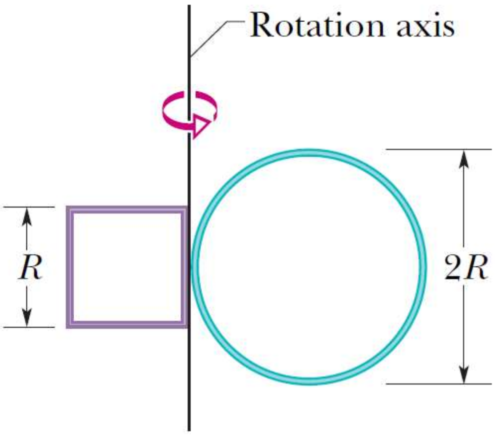
$$\omega^2 = v^2/R^2 = gr/R^2,$$

$$I_{com} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1] .$$

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

Problem 6

The figure shows a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming $R = 0.50$ m and $m = 2.0$ kg, calculate (a) the structure's rotational inertia about the axis of rotation and (b) its angular momentum about that axis.



圖中顯示了一個剛體結構，該結構由半徑為R和質量m的圓環和由四個細條組成的正方形，每個細條的長度為R和質量m。 剛體繞垂直軸以恆定速度旋轉，旋轉週期為2.5 s。 假設$R = 0.50$ m且$m = 2.0$ kg，則計算(a)剛體圍繞旋轉軸的轉動慣量，以及(b)剛體圍繞該軸的角動量。 (02小題)

(a)I=_____ kg.m^2

11: **ANS:=<u>1.6</u>**

(b)angular momentum, L=______ kg.m^2/s

12: **ANS:=<u>4.0</u>**

Solution:

For the hoop,

Of the thin bars (in the form of a square), the member along the rotation axis has no rotational inertia about that axis

$$I_1 = I_{com} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

$$I_2 = I_{com} + mh^2 = 0 + mR^2 = mR^2 .$$

$$I_3 = I_{com} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2$$

$I_3 = I_4$

the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\mathrm{kg\cdot m^2}$$

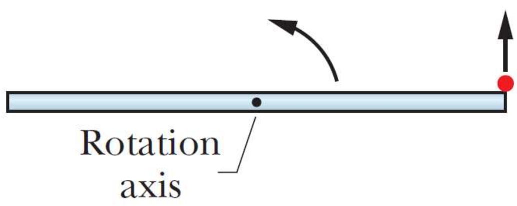
(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\mathrm{rad/s}.$$

Thus, $L = I_{total}\omega = 4.0\,\mathrm{kg\cdot m^2/s}$.

Problem 7

The figure is an overhead view of a thin uniform rod of length 0.800 m and mass M rotating horizontally at angular speed 20.0 rad/s about an axis through its center. A particle of mass M/3 initially attached to one end is ejected from the rod and travels along a path that is perpendicular to the rod at the instant of ejection. If the particle's speed v_p is 6 m/s greater than the speed of the rod end just after ejection, what is the value of v_p?



圖為長度0.8 m，質量M的細均勻棒的俯視圖，該棒繞著穿過其中心的軸以20.0 rad/s的角速率水平旋轉。 最初附著在一端的質量為M/3的粒子從桿中彈出，並在彈出時沿著垂直於桿的路徑行進。如果粒子的速度v_p比剛彈出後桿端的速度大6 m/s，那麼v_p的值是多少？ (01小題)

v_p=_____ m/s

13: **ANS:=<u>11</u>**

Solution:

By angular momentum conservation

$$L'_p + L'_r = L_p + L_r$$

$$\left(\frac{L}{2}\right) m v_p + \frac{1}{12} M L^2 \omega' = I_p\, \omega + \frac{1}{12} M L^2\, \omega$$

$$I_p = m(L/2)^2$$

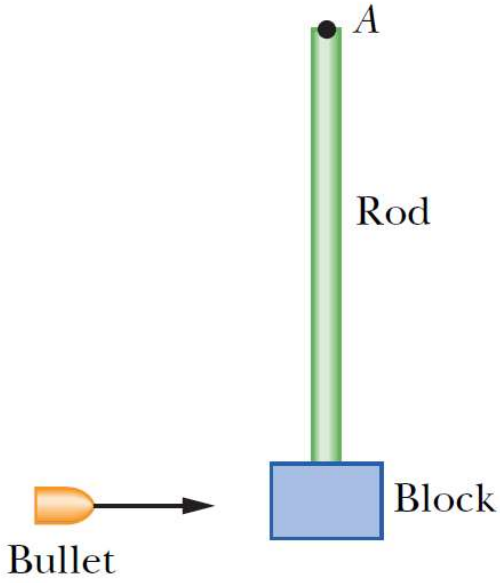
$$\omega' = v_{end}/r = (v_p - 6)/(L/2),$$

$$M = 3m \text{ and } \omega = 20 \text{ rad/s},$$

$$v_p = 11.0 \text{ m/s}.$$

Problem 8

In the figure, a 1.0 g bullet is fired into a 0.50 kg block attached to the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block-rod-bullet system then rotates in the plane of the figure, about a fixed axis at A. The rotational inertia of the rod alone about that axis at A is 0.060 kg.m^2. Treat the block as a particle. (a) What then is the rotational inertia of the block-rod-bullet system about point A? (b) If the angular speed of the system about A just after impact is 4.5 rad/s, what is the bullet's speed just before impact?



在圖中，將1.0 g子彈發射到0.50 kg塊中，該塊連接到質量為0.50 kg，0.60 m非均勻棒的末端。 然後，塊-桿-子彈系統繞A處的固定軸在圖平面中旋轉。桿子繞軸的轉動慣量為0.06 kg.m^2。 將塊視為粒子。 (a)，塊-桿-子彈系統圍繞點A的轉動慣量是多少？ (b)如果系統在撞擊後對過A軸的轉動角速率為4.5 rad/s，子彈在撞擊前的速度是多少？ (02小題)

(a)I=____ kg.m^2

14: **ANS:=0.24**

(b)bullet's speed just before impact, v_b=____ m/s

15: **ANS:=1800**

Solution:

(a) With $r = 0.60$ m, we obtain $I = 0.060 + (0.501)r^2 = 0.24\ \text{kg}\cdot\text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \Rightarrow mv_0 r = I\omega \Rightarrow (0.001)v_0(0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.