GPN1-QR1

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Problem 1

Answer the following questions: (05小題)

(a)Certain neutron stars (extremely dense stars) are believed to be rotating at about 2 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?
M_{min}=_____ kg

01: **ANS:=<u>9.4E24</u>**

(b)The satellite of a planet travels in an approximately circular orbit of radius 9.4E6 m with a period of 2.754E4 s. Calculate the mass of the planet from this information.
mass M=______ kg

02: **ANS:=<u>6.5E23</u>**

(c)A planet of mass 6E24 kg and radius 6.4E6 m, the gravitational acceleration at a distance 1.6E6 m from its center=_______ m/s^2 (Assume the mass distribution is uniform.)

03: **ANS:=<u>2.45</u>**

(d)A planet is a distance r from the star which it is orbiting with a period of T. Suppose there is another planet of distance $10r$ from the star. Assume both orbits are approximately circular. The period of the distance planet=______ T

04: **ANS:=<u>31.6</u>**

(e)A loudspeaker produces a musical sound by means of the oscillation of a diaphragm whose amplitude is limited to 2.0 μm. At what frequency is the magnitude a of the diaphragm's acceleration equal to g?
frequency f=______ Hz

05: **ANS:=<u>352.3</u>**

Problem 2

有一個斜角45度的斜面，從斜面高10公尺的地方將一個圓環釋放，圓環的質量1公斤，半徑2公尺，如果這是一個平滑的斜面沒有摩擦，請計算(a)圓環從靜止釋放，到達底部的速度？(b)到達底部所需要的時間？如果這個斜面存在摩擦，使得圓環從靜止開始滾動而沒有滑動地運動到斜面底部，請計算在這個情況下，(c)圓環質心運動的加速度？(d)從靜止釋放到達底部所需要的時間，(e)到達底部時的質心速度。(f)請計算斜面的摩擦力。(g)斜面與圓環之間的摩擦係數的最小值為何。 (07小題)

(a)沒有摩擦，圓環到達底部的速度=______ m/s

06: **ANS:=14.0**

(b)沒有摩擦，圓環到達底部所需要的時間=______ s

07: **ANS:=2.02**

(c)滾動時圓環質心運動的加速度=______ m/s^2

08: **ANS:=3.465**

(d)從靜止釋放滾動到達底部所需要的時間=______ s

09: **ANS:=2.86**

(e)從靜止釋放滾動到達底部的速度=______ m/s

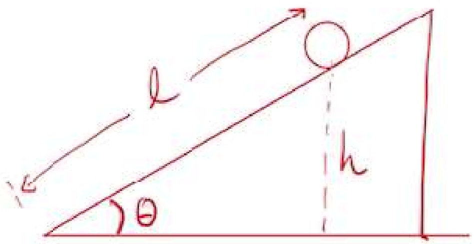
10: **ANS:=9.90**

(f)斜面的摩擦力=______ N

11: **ANS:** 3.465

(g)斜面與圓環之間的摩擦係數的最小值=______

12: **ANS:=** 0.500



圓環 : M, R

$I = MR^2$

$l = \frac{h}{\sin\theta}$

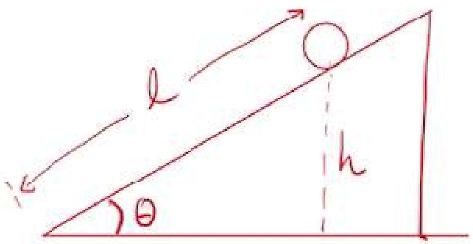
M = 1

R = 2

no friction $\Rightarrow a = g\sin\theta = 9.8 \times \frac{1}{\sqrt{2}} = 6.93$

$v = \sqrt{2al} = \sqrt{2(6.93)(14.14)} = 14.0$ (m/s)

$t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{2(14.14)}{6.93}} = 2.02$ (s)



圓環 : M, R

$I = MR^2$

$\ell = \dfrac{h}{\sin\theta}$

$f_s = Ma = 13.465$

$f_{s,max} = \mu_s N = \mu_s (Mg\cos\theta) > 3.465$

$\mu_s > \dfrac{3.465}{1(9.8)(\frac{1}{\sqrt{2}})} =$ 0.500

$Mg\sin\theta - f_s = Ma$

$M = 1$
$R = 2$

$R f_s = I\alpha, \; f_s = MR\alpha = Ma$

$Mg\sin\theta = 2Ma, \quad a = \frac{1}{2} g \sin\theta = \frac{1}{2\sqrt{2}} g$
$= 3.465$

$\ell = \dfrac{10}{\frac{1}{\sqrt{2}}} = 10\sqrt{2} = 14.14$

$v^2 = 2a\ell = 2(3.465)(14.14) = 98.0$

$v = 9.90 \; (m/s)$

$\ell = \frac{1}{2} a t^2, \quad t = \sqrt{\dfrac{2\ell}{a}} = \sqrt{\dfrac{2(14.14)}{3.465}} = 2.86 \; (s)$

(f)斜面的摩擦力=_______ N

11: **ANS:** 3.465

(g)斜面與圓環之間的摩擦係數的最小值=______

12: **ANS:=** 0.500

Problem 3

有一個質量m的物體受到一個彈性係數為k的彈簧作用，而進行簡諧運動。

- (a)請寫下這個物體所要滿足的牛頓運動第二定律的方程式：$\frac{d^2x}{dt^2} =$______.
- (b)在這個方程式中加速度請用時間的二次微分來表示。假設這個物體在初始時間的位置為A，速度為0，請證明這個物體的位置對時間的函數$x(t) = A\cos(bt)$滿足(a)的微分方程式。
- (c)$x(t)$當中的b與k, m的關係。
- (d)在簡諧運動中物體的週期T。

(a) $\frac{d^2x}{dt^2} =$______ [k,m,x]

13: **ANS:=-(k/m)*x**

(b)加速度$a(t)$=______ [t,A,k,m]

14: **ANS:=-(k/m)*A*cos(sqrt(k/m)*t)**

(c)b=______ [k,m,A]

15: **ANS:=sqrt(k/m)**

(d)週期T=______ [k,m,A]

16: **ANS:=2*pi*sqrt(m/k)**

$F = ma = m\frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} = \frac{F}{m} = -\frac{k}{m}x$ — (1)

$x(t) = A\cos(bt)$

$\frac{dx}{dt} = -bA\sin(bt)$

$\frac{d^2x}{dt^2} = -b^2A\cos(bt) = -b^2x$ — (2)

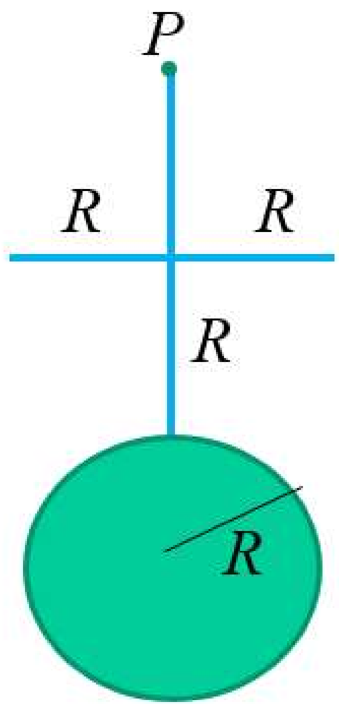
比較 (1) & (2) $b^2 = \frac{k}{m} \Rightarrow b = \sqrt{\frac{k}{m}} = \omega$

$a(t) = -\left(\frac{k}{m}\right)A\cos\left(\sqrt{\frac{k}{m}}\,t\right)$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Problem 4

The cross formed by 2 identical rods (both are of mass m and length $2R$) is connected by a disk of mass m and radius R as shown in the figure. (a)Calculate the moment of inertia of the two rods with the axis passing through P. (b)Calculate the moment of inertia of the disk with the axis passing through P. (c)Calculate the distance of the center of mass of the whole system(cross plus disk) to the pivot point P. (d)Find the period of the rigid body as a physical pendulum with P as the pivot point. (剛體的 3 個物體，2 個桿和一個圓盤，質量均為 m。) (04小題)



(a)the moment of inertia of the two rods with the axis passing through P, $I_{rod,P}$=______ kg.m^2

<u>17:</u> **ANS:=<u>85.33</u>**

(b)the moment of inertia of the disk with the axis passing through P, $I_{disk,P}$ =______ kg.m^2

<u>18:</u> **ANS:=<u>304</u>**

(c)the distance of the center of mass to the pivot point P, h=____ m

<u>19:</u> **ANS:=<u>6.667</u>**

(d)T=______ s

<u>20:</u> **ANS:=<u>6.26</u>**

$$I_{rod,P} = \frac{1}{3} m(2R)^2 + \frac{1}{12} m(2R)^2 + mR^2$$

$m = 2$

$R = 4$

$$= \left(\frac{4}{3} + \frac{4}{12} + 1\right) mR^2 = \frac{8}{3} mR^2 = 85.33$$

$$I_{disk,P} = \frac{1}{2} mR^2 + m(3R)^2 = \frac{19}{2} mR^2 = 304$$

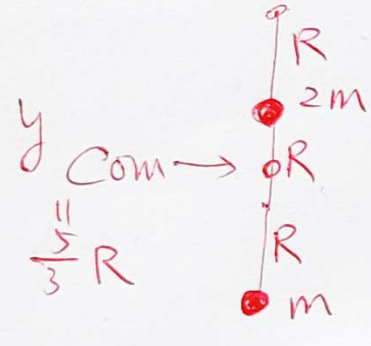
$$I_P = \left(\frac{8}{3} + \frac{19}{2}\right) mR^2 = \frac{73}{6} mR^2 \cong 12.17\, mR^2$$

$$y_{com} = \frac{5}{3} R = 6.667$$

$M = 3m$

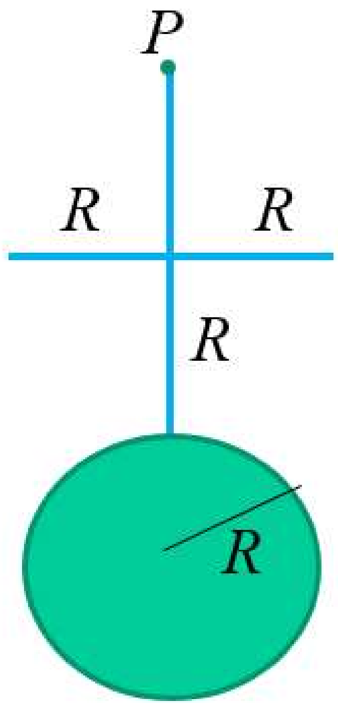
$$T = 2\pi \sqrt{\frac{I_P}{Mgh}} = 2\pi \sqrt{\frac{12.17 mR^2}{3mg\left(\frac{5}{3}R\right)}}$$

$$= 2\pi \sqrt{\frac{12.17R}{5g}} = 6.26 \text{ (s)}$$



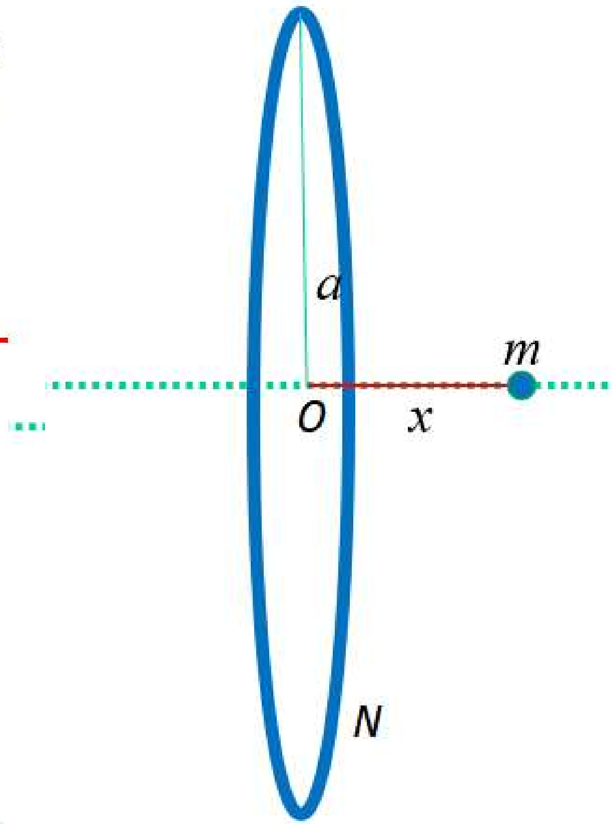
$$y_{com} = \frac{2mR + m(3R)}{3m}$$

$$= \frac{5}{3} R = h$$



Problem 5

A ring of mass N and radius a is fixed in space. A particle of mass m lying on the axis(x-axis) through the center of the ring (see figure) is attracted by the gravitational force of the ring. The origin of x-coordinate O is at the center of the ring. Answer the following questions. (04小題)



(a)Find the potential energy of the system as a function of x, $U(x)$=____ [G,N,m,a,x], x is the x-coordinate of m. Be careful of the sign.

21: **ANS:=-(G*N*m)/sqrt(x**2+a**2)**

(b)The force $F(x)$ on m can be derived by the differentiation of $U(x$, $F(x)$ =_____ [G,N,m,a,x]

22: **ANS:=-(G*N*m)/(x**2+a**2)**(3/2)*x**

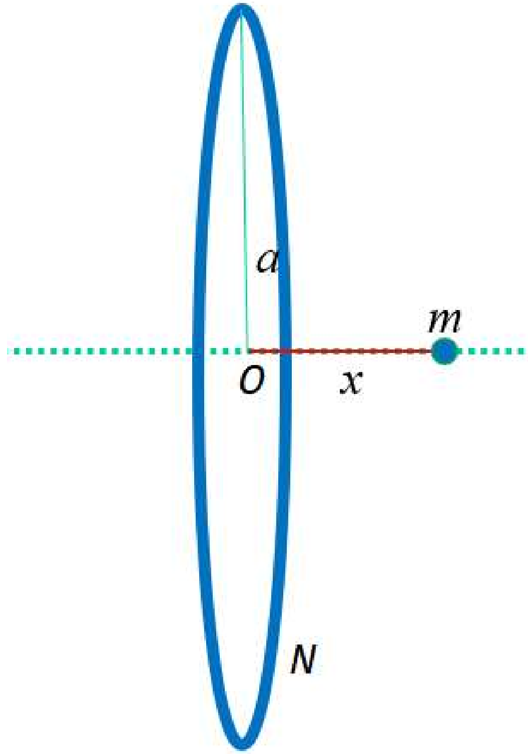
(c)Suppose m is initially at $x = a$ and zero speed. Find the speed of m when it is attracted to by ring to the center of the ring. v=____ [G,N,m,a]

23: **ANS:=sqrt((2-sqrt(2))*G*N/a)**

(d)For m to be very close to the center, the force is linear in x. So the motion can be taken as SHM. The period of the motion=_____ [G,m,a]

24: **ANS:=2*pi*sqrt(a**3/(G*N))**

所有在圆環上的質量点与 m 的距離均為 $\sqrt{x^2+a^2}$
因此 m 得之於環的重力位能 $U(x)$ 等同單一質点 質量 $=100m$ (環的整体質量) 給予 m 的位能



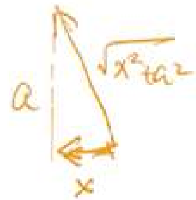
$$U(x) = -\frac{GNm}{\sqrt{x^2+a^2}}$$

$$F(x) = -\frac{dU}{dx} = GNm\left(-\frac{1}{2}\right)(x^2+a^2)^{-\frac{3}{2}}(2x)$$

$$= -\frac{GNm}{(x^2+a^2)^{\frac{3}{2}}}\,x$$

From Newton's law:

$$F_x = -\frac{GNm}{x^2+a^2}\,\frac{x}{\sqrt{x^2+a^2}}$$



$$= -\frac{GNm}{(x^2+a^2)^{3/2}}\,x$$

$$x \ll a \Rightarrow F_x \cong -\underbrace{\frac{GNm}{a^3}}_{k}\,x$$

$$T = 2\pi\sqrt{\frac{m}{\frac{GNm}{a^3}}} = 2\pi\sqrt{\frac{a^3}{GN}}$$

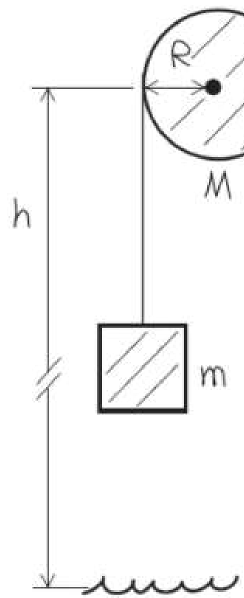
$$\Delta U = U(0) - U(a) = -\frac{GNm}{a} - \left(-\frac{GNm}{\sqrt{2}a}\right)$$

$$= -\left(\frac{1}{a} - \frac{1}{\sqrt{2}a}\right)GNm \qquad \Delta K = -\Delta U = \left(1 - \frac{1}{\sqrt{2}}\right)\frac{GNm}{a}$$

$$\frac{1}{2}mv^2 = \Delta K \Rightarrow v = \left[(2-\sqrt{2})\frac{GN}{a}\right]^{1/2}$$

Problem 6

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass $m = M/4$ and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. What are (a)the acceleration of the falling block and (b)the tension in the cable? (c)Find the speed of the falling block as the block strikes the floor. (d)Find the time for the block to hit the floor. (e)Find the angular momentum of the disk with respect to the axle of the cylinder when the block is at the height of $h/2$. (05小題)



(a)the acceleration a=______ [g,M,R,h]

25: **ANS:=g/3**

(b)the tension T=_______ [g,M,R,h]

26: **ANS:=M*g/6**

(c)the speed v=_______ [g,M,R,h]

27: **ANS:=sqrt((2*g*h)/3)**

(d)the time t=_______ [g,M,R,h]

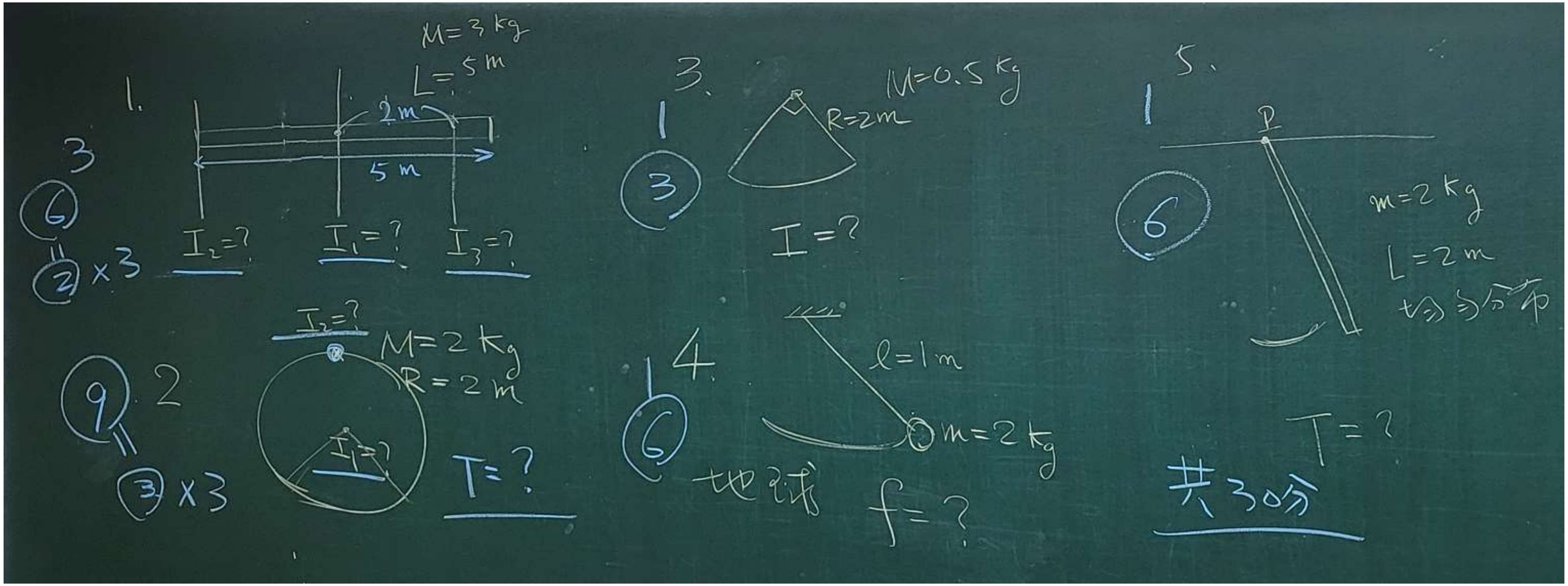
28: **ANS:=sqrt((6*h/g)**

(e)the angular momentum=_ [g,M,R,h]

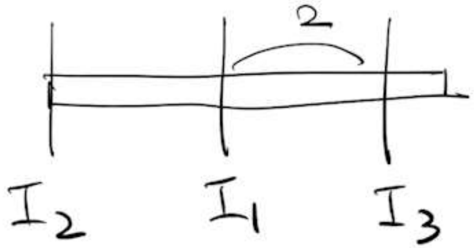
29: **ANS:=1/2*M*R*sqrt(g*h/3)**

$$mg - T = ma$$
$$TR = I\alpha = \frac{1}{2}MR^2\alpha = \frac{1}{2}MRa$$
$$T = \frac{1}{2}Ma$$
$$mg - \frac{1}{2}Ma = ma$$
$$\left(\frac{1}{2}M + m\right)a = mg$$
$$\left(\frac{1}{2}M + \frac{1}{4}M\right)a = \frac{1}{4}Mg$$
$$\frac{3}{4}a = \frac{1}{4}g$$
$$a = \frac{1}{3}g$$
$$T = \frac{1}{2}Ma = \frac{1}{6}Mg$$

$$v^2 = v_0^2 + 2ah$$
$$= 0 + 2\left(\frac{g}{3}\right)h$$
$$v = \sqrt{\frac{2gh}{3}}$$
$$h = \frac{1}{2}at^2$$
$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h}{\frac{g}{3}}} = \sqrt{\frac{6h}{g}}$$
$$v'^2 = 2\left(\frac{g}{3}\right)\left(\frac{h}{2}\right) \qquad v' = \sqrt{\frac{gh}{3}}$$
$$L = I\omega = \frac{1}{2}MR^2\left(\frac{v'}{R}\right)$$
$$= \frac{1}{2}MR\sqrt{\frac{gh}{3}}$$



1. $M=3$, $L=5$

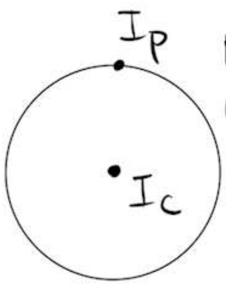


$$I_1 = \frac{1}{12}(3)(5)^2 = 6.25$$

$$I_2 = \frac{1}{3}(3)(5)^2 = 25$$

$$I_3 = 6.25 + 3(2)^2 = 18.25$$

2.



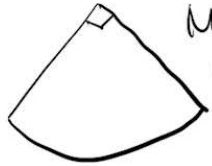
$M=2$
$R=2$

$$I_c = \frac{1}{2}MR^2 = \frac{1}{2}(2)(2)^2 = 4$$

$$I_p = 4 + 2(2)^2 = 12$$

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{12}{2(9.8)(2)}} = 3.48\ (s)$$

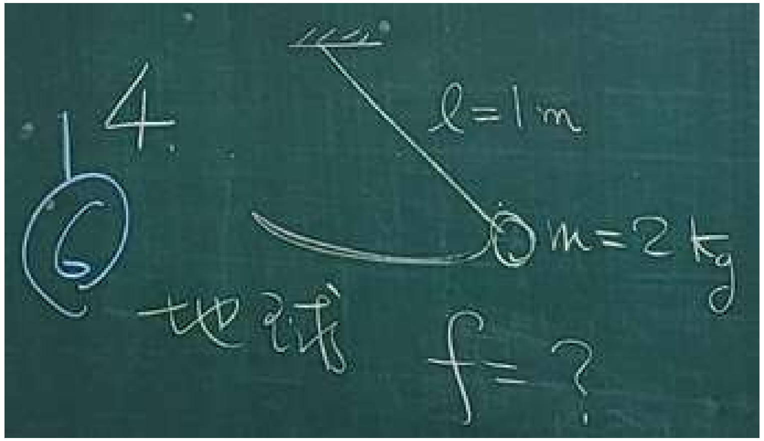
3.



$M=0.5$
$R=2$

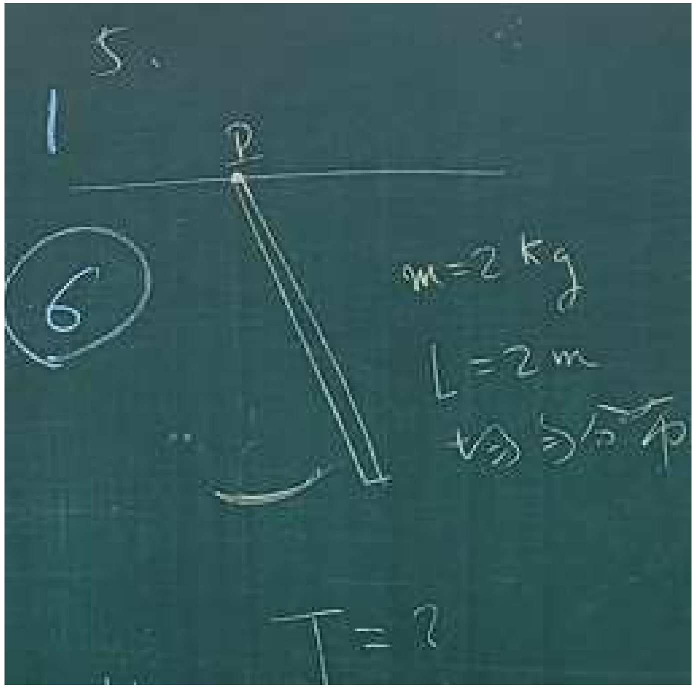
$$I_4 = \frac{1}{2}MR^2 = I_c = 4$$

$$I = \frac{1}{4}I_4 = 1$$



4. $T = 2\pi\sqrt{\frac{1}{9.8}} = 2.007$

$f = \frac{1}{T} = 0.498$



5. $I_P = \frac{1}{3}(2)(2)^2$

$= \frac{8}{3}$

$T = 2\pi\sqrt{\frac{I_P}{mgh}}$

$= 2\pi\sqrt{\frac{\frac{8}{3}}{2(9.8)(1)}}$

$= 2.32(s)$